Probability

Bernoulli Distribution

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Probability Density Function (PDF)

- **Statistical inference** involves **drawing a sample from a population** and **analyzing** the sample data to learn about the population.
- In many situations, **one has an approximate knowledge** of the probability mass function or probability density function of the **population**. In these cases, the probability mass or density function can often be well **approximated by one of several standard families** of curves, or functions.
- In this chapter, we **describe some of these standard functions**, and for each one we describe **some conditions under which** it is appropriate

Bernoulli Distribution

- Imagine an experiment that can **result in one of two outcomes**. One outcome is labeled **success**, and the other outcome is labeled **failure**.
- The **probability of success is denoted by $\boldsymbol{p}$**. The probability of failure is therefore $1 - p$. Such a trial is called a Bernoulli trial with success probability p
  - Toss of a coin
  - accept / reject component

Bernoulli Distribution

- For any Bernoulli trial, we define a **random variable** $\boldsymbol{X}$ as follows: If the experiment results in **success**, then **X = 1**. Otherwise, **X = 0**. It follows that X is a discrete random variable, with probability mass function p(x) defined by
  - $p(0) = P(X = 0) = 1 - p$
  - $p(1) = P(X = 1) = p$

Bernoulli Distribution

- The random variable X is said to have the **Bernoulli distribution with parameter p**.
- The notation is **X ~ Bernoulli(p)**

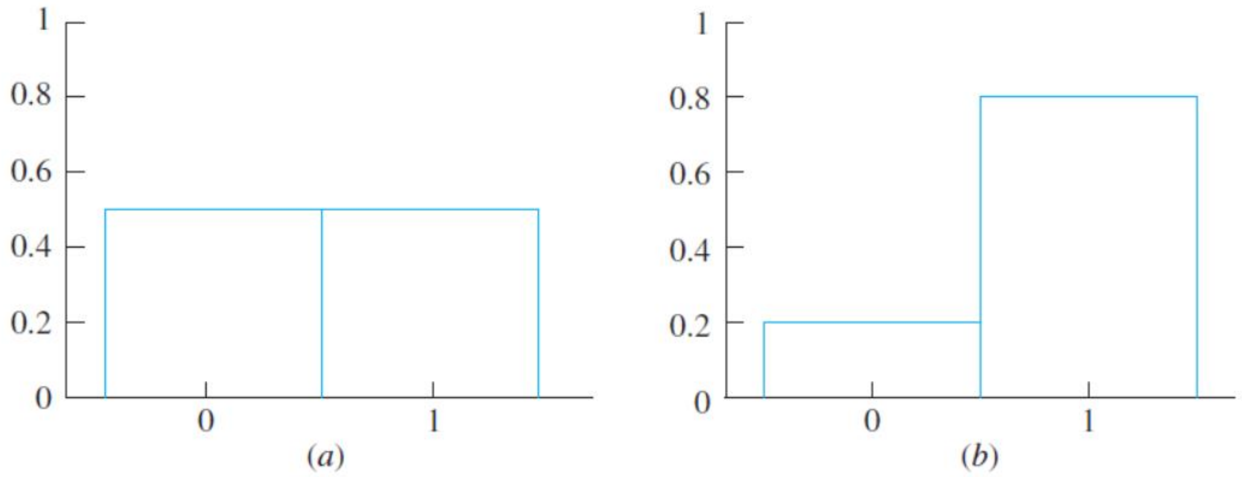

FIGURE 4.1 *(a)* The Bernoulli(0.5) probability histogram. *(b)* The Bernoulli(0.8) probability histogram.

Bernoulli Distribution

- Consider whether the next person buying a computer at a university book store buys a laptop or a desktop model. Let
- $X = \begin{cases} 1 & \text{if the customer purchases a laptop computer} \\ 0 & \text{if the customer purchases a desktop computer} \end{cases}$
- If 20% of all purchasers during that week select a laptop, the PMF for X is
  - $p(\mathbf{0}) = P(X = 0) =$ P(next customer purchases a **desktop** model) = .8
  - $p(\mathbf{1}) = P(X = 1) =$ P(next customer purchases a **laptop** model) =.2
  - $p(x) = P(X = x) = 0\ for\ x \neq 0\ or\ 1$

Bernoulli Distribution

- Consider whether the next person buying a computer at a university bookstore buys a laptop or a desktop model.
- $p(x) = \begin{cases} 0.8 & if\ x = 0 \\ 0.2 & if\ x = 1 \\ 0 & x \neq 0\ or\ 1 \end{cases}$

Example - 01

- A coin has probability 0.5 of landing heads when tossed. Let X = 1 if the coin comes up heads, and X = 0 if the coin comes up tails. What is the distribution of X?
- Since X = 1 when heads comes up, heads is the success outcome. The success probability, P(X = 1), is equal to 0.5. Therefore X $\sim$ Bernoulli(0.5).

Example - 02

- A die has probability 1/6 of coming up 6 when rolled. Let $X = 1$ if the die comes up 6, and $X = 0$ otherwise. What is the distribution of X?
- The success probability is $p = P(X = 1) = 1/6$. Therefore $X \sim$ Bernoulli(1/6).

Example - 03

- Ten percent of the components manufactured by a certain process are defective. A component is chosen at random. Let $X = 1$ if the component is defective, and $X = 0$ otherwise. What is the distribution of X?
- The success probability is $p = P(X = 1) = 0.1$. Therefore $X \sim \text{Bernoulli}(0.1)$.

Mean and Variance

- It is easy to compute the mean and variance of a Bernoulli random variable. If $X \sim Bernoulli(p)$, then
- $\mu_x = (0)(1-p) + (1)(p) = p$
- $\sigma_x^2 = (0-p)^2(1-p) + (1-p)^2(p) = p(1-p)$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy