

Probability

Binomial Distribution – Solved Examples

Prof. Dr. Mostafa Elhosseini

<https://youtube.com/drmelhosseini>

| Conditions for Binomial Distribution

- **Each of n trials has two possible outcomes.** The **outcome of interest** is called a **success** and the other outcome is called a failure.
- **Each trial has the same probability of a success.** This is denoted by p , so the probability of a success is p and the probability of a failure is $1 - p$.
- **The n trials are independent.** That is, the result for one trial does not depend on the results of other trials.

The binomial random variable **X is the number of successes in the n trials**

X has the binomial distribution with parameters n and p , denoted $X \sim \text{Bin}(n, p)$.

| Binomial Distribution PMF

- If $X \sim \mathbf{Bin}(n, p)$, the probability mass function of X is

$$p(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

- n : # independent Bernoulli Trials
- p : Success probability
- $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ binomial coefficient = nCx

| Example - 02

- A **lot** contains several thousand components, **10%** of which are **defective**. **Seven** components are sampled from the lot. Let **X** represent the **number of defective components** in the sample. **What is the distribution of X?**
- Since the **sample size is small compared to the population** (i.e., less than 5%), the number of successes in the sample approximately follows a binomial distribution. Therefore we model X with the **Bin(7, 0.1)** distribution.

| Example - 02

- Find the **probability mass function** of the random variable X if $X \sim \text{Bin}(10, 0.4)$. Find $P(X = 5)$.
- $p(x) = \binom{10}{x} 0.4^x (0.6)^{10-x}$
- $p(5) = \binom{10}{5} 0.4^5 (0.6)^5 = 0.2007$

| Example - 03

- A fair die is rolled eight times. Find the probability that no more than 2 sixes come up
- Each roll of the die is a Bernoulli trial with success probability $1/6$
- Let X denote the number of sixes in 8 rolls Then $X \sim \text{Bin}(8, 1/6)$.
- We need to find $P(X \leq 2)$. Using the probability mass function
- $P(X \leq 2) = P(X = 0 \text{ or } X = 1 \text{ or } X = 2)$
- $P(X = 0) + P(X = 1) + P(X = 2) = 0.8652$

TABLE A.1 Cumulative binomial distribution

$$F(x) = P(X \leq x) = \sum_{k=0}^x \frac{n!}{k!(n-k)!} p^k (1-p)^{(n-k)}$$

n	x	p												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
2	0	0.902	0.810	0.640	0.562	0.490	0.360	0.250	0.160	0.090	0.062	0.040	0.010	0.003
	1	0.997	0.990	0.960	0.938	0.910	0.840	0.750	0.640	0.510	0.438	0.360	0.190	0.098
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0	0.857	0.729	0.512	0.422	0.343	0.216	0.125	0.064	0.027	0.016	0.008	0.001	0.000
	1	0.993	0.972	0.896	0.844	0.784	0.648	0.500	0.352	0.216	0.156	0.104	0.028	0.007
	2	1.000	0.999	0.992	0.984	0.973	0.936	0.875	0.784	0.657	0.578	0.488	0.271	0.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	0	0.815	0.656	0.410	0.316	0.240	0.130	0.062	0.026	0.008	0.004	0.002	0.000	0.000
	1	0.986	0.948	0.819	0.738	0.652	0.475	0.313	0.179	0.084	0.051	0.027	0.004	0.000
	2	1.000	0.996	0.973	0.949	0.916	0.821	0.688	0.525	0.348	0.262	0.181	0.052	0.014
	3	1.000	1.000	0.998	0.996	0.992	0.974	0.938	0.870	0.760	0.684	0.590	0.344	0.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0	0.774	0.590	0.328	0.237	0.168	0.078	0.031	0.010	0.002	0.001	0.000	0.000	0.000
	1	0.977	0.919	0.737	0.633	0.528	0.337	0.187	0.087	0.031	0.016	0.007	0.000	0.000
	2	0.999	0.991	0.942	0.896	0.837	0.683	0.500	0.317	0.163	0.104	0.058	0.009	0.00
	3	1.000	1.000	0.993	0.984	0.969	0.913	0.812	0.663	0.472	0.367	0.263	0.081	0.023
	4	1.000	1.000	1.000	0.999	0.998	0.990	0.969	0.922	0.832	0.763	0.672	0.410	0.226
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	0	0.735	0.531	0.262	0.178	0.118	0.047	0.016	0.004	0.001	0.000	0.000	0.000	0.000
	1	0.967	0.886	0.655	0.534	0.420	0.233	0.109	0.041	0.011	0.005	0.002	0.000	0.000
	2	0.998	0.984	0.901	0.831	0.744	0.544	0.344	0.179	0.070	0.038	0.017	0.001	0.000
	3	1.000	0.999	0.983	0.962	0.930	0.821	0.656	0.456	0.256	0.169	0.099	0.016	0.002
	4	1.000	1.000	0.998	0.995	0.989	0.959	0.891	0.767	0.580	0.466	0.345	0.114	0.033
	5	1.000	1.000	1.000	1.000	0.999	0.996	0.984	0.953	0.882	0.822	0.738	0.469	0.265
	6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	0	0.698	0.478	0.210	0.133	0.082	0.028	0.008	0.002	0.000	0.000	0.000	0.000	0.000
	1	0.956	0.850	0.577	0.445	0.329	0.159	0.063	0.019	0.004	0.001	0.000	0.000	0.000
	2	0.996	0.974	0.852	0.756	0.647	0.420	0.227	0.096	0.029	0.013	0.005	0.000	0.000
	3	1.000	0.997	0.967	0.929	0.874	0.710	0.500	0.290	0.126	0.071	0.033	0.003	0.000
	4	1.000	1.000	0.995	0.987	0.971	0.904	0.773	0.580	0.353	0.244	0.148	0.026	0.004
	5	1.000	1.000	1.000	0.999	0.996	0.981	0.938	0.841	0.671	0.555	0.423	0.150	0.044
	6	1.000	1.000	1.000	1.000	1.000	0.998	0.992	0.972	0.918	0.867	0.790	0.522	0.302
	7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE A.1 Cumulative binomial distribution (continued)

n	x	p												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
8	0	0.663	0.430	0.168	0.100	0.058	0.017	0.004	0.001	0.000	0.000	0.000	0.000	0.000
	1	0.943	0.813	0.503	0.367	0.255	0.106	0.035	0.009	0.001	0.000	0.000	0.000	0.000
	2	0.994	0.962	0.797	0.679	0.552	0.315	0.145	0.050	0.011	0.004	0.001	0.000	0.000
	3	1.000	0.995	0.944	0.886	0.806	0.594	0.363	0.174	0.058	0.027	0.010	0.000	0.000
	4	1.000	1.000	0.990	0.973	0.942	0.826	0.637	0.406	0.194	0.114	0.056	0.005	0.000
	5	1.000	1.000	0.999	0.996	0.989	0.950	0.855	0.685	0.448	0.321	0.203	0.038	0.006
	6	1.000	1.000	1.000	1.000	0.999	0.991	0.965	0.894	0.745	0.633	0.497	0.187	0.057
	7	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.983	0.942	0.900	0.832	0.570	0.337
9	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0	0.630	0.387	0.134	0.075	0.040	0.010	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.929	0.775	0.436	0.300	0.196	0.071	0.020	0.004	0.000	0.000	0.000	0.000	0.000
	2	0.992	0.947	0.738	0.601	0.463	0.232	0.090	0.025	0.004	0.001	0.000	0.000	0.000
	3	0.999	0.992	0.914	0.834	0.730	0.483	0.254	0.099	0.025	0.010	0.003	0.000	0.000
	4	1.000	0.999	0.980	0.951	0.901	0.733	0.500	0.267	0.099	0.049	0.020	0.001	0.000
	5	1.000	1.000	0.997	0.990	0.975	0.901	0.746	0.517	0.270	0.166	0.086	0.008	0.001
	6	1.000	1.000	1.000	0.999	0.996	0.975	0.910	0.768	0.537	0.399	0.262	0.053	0.008
10	7	1.000	1.000	1.000	1.000	1.000	0.996	0.980	0.929	0.804	0.700	0.564	0.225	0.071
	8	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.990	0.960	0.925	0.866	0.613	0.370
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0	0.599	0.349	0.107	0.056	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.914	0.736	0.376	0.244	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000	0.000
	2	0.988	0.930	0.678	0.526	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000	0.000
	3	0.999	0.987	0.879	0.776	0.650	0.382	0.172	0.055	0.011	0.004	0.001	0.000	0.000
	4	1.000	0.998	0.967	0.922	0.850	0.633	0.377	0.166	0.047	0.020	0.006	0.000	0.000
11	5	1.000	1.000	0.994	0.980	0.953	0.834	0.623	0.367	0.150	0.078	0.033	0.002	0.000
	6	1.000	1.000	0.999	0.996	0.989	0.945	0.828	0.618	0.350	0.224	0.121	0.013	0.001
	7	1.000	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.474	0.322	0.070	0.012
	8	1.000	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.756	0.624	0.264	0.086
	9	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.944	0.893	0.651	0.401
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0	0.569	0.314	0.086	0.042	0.020	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.898	0.697	0.322	0.197	0.113	0.030	0.006	0.001	0.000	0.000	0.000	0.000	0.000
12	2	0.985	0.910	0.617	0.455	0.313	0.119	0.033	0.006	0.001	0.000	0.000	0.000	0.000
	3	0.998	0.981	0.839	0.713	0.570	0.296	0.113	0.029	0.004	0.001	0.000	0.000	0.000
	4	1.000	0.997	0.950	0.885	0.790	0.533	0.274	0.099	0.022	0.008	0.002	0.000	0.000
	5	1.000	1.000	0.988	0.966	0.922	0.753	0.500	0.247	0.078	0.034	0.012	0.000	0.000
	6	1.000	1.000	0.998	0.992	0.978	0.901	0.726	0.467	0.210	0.115	0.050	0.003	0.000
	7	1.000	1.000	1.000	0.999	0.996	0.971	0.887	0.704	0.430	0.287	0.161	0.019	0.002
	8	1.000	1.000	1.000	1.000	0.999	0.994	0.967	0.881	0.687	0.545	0.383	0.090	0.015
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE A.1 Cumulative binomial distribution (continued)

<i>n</i>	<i>x</i>	<i>p</i>												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
11	9	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.970	0.887	0.803	0.678	0.303	0.102
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.980	0.958	0.914	0.686	0.431
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
12	0	0.540	0.282	0.069	0.032	0.014	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.882	0.659	0.275	0.158	0.085	0.020	0.003	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.980	0.889	0.558	0.391	0.253	0.083	0.019	0.003	0.000	0.000	0.000	0.000	0.000
	3	0.998	0.974	0.795	0.649	0.493	0.225	0.073	0.015	0.002	0.000	0.000	0.000	0.000
	4	1.000	0.996	0.927	0.842	0.724	0.438	0.194	0.057	0.009	0.003	0.001	0.000	0.000
	5	1.000	0.999	0.981	0.946	0.882	0.665	0.387	0.158	0.039	0.014	0.004	0.000	0.000
	6	1.000	1.000	0.996	0.986	0.961	0.842	0.613	0.335	0.118	0.054	0.019	0.001	0.000
	7	1.000	1.000	0.999	0.997	0.991	0.943	0.806	0.562	0.276	0.158	0.073	0.004	0.000
	8	1.000	1.000	1.000	1.000	0.998	0.985	0.927	0.775	0.507	0.351	0.205	0.026	0.002
	9	1.000	1.000	1.000	1.000	1.000	0.997	0.981	0.917	0.747	0.609	0.442	0.111	0.020
	10	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.980	0.915	0.842	0.725	0.341	0.118
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.986	0.968	0.931	0.718	0.460
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
13	0	0.513	0.254	0.055	0.024	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.865	0.621	0.234	0.127	0.064	0.013	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.975	0.866	0.502	0.333	0.202	0.058	0.011	0.001	0.000	0.000	0.000	0.000	0.000
	3	0.997	0.966	0.747	0.584	0.421	0.169	0.046	0.008	0.001	0.000	0.000	0.000	0.000
	4	1.000	0.994	0.901	0.794	0.654	0.353	0.133	0.032	0.004	0.001	0.000	0.000	0.000
	5	1.000	0.999	0.970	0.920	0.835	0.574	0.291	0.098	0.018	0.006	0.001	0.000	0.000
	6	1.000	1.000	0.993	0.976	0.938	0.771	0.500	0.229	0.062	0.024	0.007	0.000	0.000
	7	1.000	1.000	0.999	0.994	0.982	0.902	0.709	0.426	0.165	0.080	0.030	0.001	0.000
	8	1.000	1.000	1.000	0.999	0.996	0.968	0.867	0.647	0.346	0.206	0.099	0.006	0.000
	9	1.000	1.000	1.000	1.000	0.999	0.992	0.954	0.831	0.579	0.416	0.253	0.034	0.003
	10	1.000	1.000	1.000	1.000	1.000	0.999	0.989	0.942	0.798	0.667	0.498	0.134	0.025
	11	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.987	0.936	0.873	0.766	0.379	0.135
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.990	0.976	0.945	0.746	0.487
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
14	0	0.488	0.229	0.044	0.018	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.847	0.585	0.198	0.101	0.047	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.970	0.842	0.448	0.281	0.161	0.040	0.006	0.001	0.000	0.000	0.000	0.000	0.000
	3	0.996	0.956	0.698	0.521	0.355	0.124	0.029	0.004	0.000	0.000	0.000	0.000	0.000
	4	1.000	0.991	0.870	0.742	0.584	0.279	0.090	0.018	0.002	0.000	0.000	0.000	0.000
	5	1.000	0.999	0.956	0.888	0.781	0.486	0.212	0.058	0.008	0.002	0.000	0.000	0.000
	6	1.000	1.000	0.988	0.962	0.907	0.692	0.395	0.150	0.031	0.010	0.002	0.000	0.000
	7	1.000	1.000	0.998	0.990	0.969	0.850	0.605	0.308	0.093	0.038	0.012	0.000	0.000

TABLE A.1 Cumulative binomial distribution (continued)

n	x	p												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
14	8	1.000	1.000	1.000	0.998	0.992	0.942	0.788	0.514	0.219	0.112	0.044	0.001	0.000
	9	1.000	1.000	1.000	1.000	0.998	0.982	0.910	0.721	0.416	0.258	0.130	0.009	0.000
	10	1.000	1.000	1.000	1.000	1.000	0.996	0.971	0.876	0.645	0.479	0.302	0.044	0.004
	11	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.960	0.839	0.719	0.552	0.158	0.030
	12	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.992	0.953	0.899	0.802	0.415	0.153
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.993	0.982	0.956	0.771	0.512
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	0	0.463	0.206	0.035	0.013	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.829	0.549	0.167	0.080	0.035	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.964	0.816	0.398	0.236	0.127	0.027	0.004	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.995	0.944	0.648	0.461	0.297	0.091	0.018	0.002	0.000	0.000	0.000	0.000	0.000
	4	0.999	0.987	0.836	0.686	0.515	0.217	0.059	0.009	0.001	0.000	0.000	0.000	0.000
	5	1.000	0.998	0.939	0.852	0.722	0.403	0.151	0.034	0.004	0.001	0.000	0.000	0.000
	6	1.000	1.000	0.982	0.943	0.869	0.610	0.304	0.095	0.015	0.004	0.001	0.000	0.000
	7	1.000	1.000	0.996	0.983	0.950	0.787	0.500	0.213	0.050	0.017	0.004	0.000	0.000
	8	1.000	1.000	0.999	0.996	0.985	0.905	0.696	0.390	0.131	0.057	0.018	0.000	0.000
	9	1.000	1.000	1.000	0.999	0.996	0.966	0.849	0.597	0.278	0.148	0.061	0.002	0.000
	10	1.000	1.000	1.000	1.000	0.999	0.991	0.941	0.783	0.485	0.314	0.164	0.013	0.001
	11	1.000	1.000	1.000	1.000	1.000	0.998	0.982	0.909	0.703	0.539	0.352	0.056	0.005
	12	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.973	0.873	0.764	0.602	0.184	0.036
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.965	0.920	0.833	0.451	0.171
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.987	0.965	0.794	0.537
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
16	0	0.440	0.185	0.028	0.010	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.811	0.515	0.141	0.063	0.026	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.957	0.789	0.352	0.197	0.099	0.018	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.993	0.932	0.598	0.405	0.246	0.065	0.011	0.001	0.000	0.000	0.000	0.000	0.000
	4	0.999	0.983	0.798	0.630	0.450	0.167	0.038	0.005	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.997	0.918	0.810	0.660	0.329	0.105	0.019	0.002	0.000	0.000	0.000	0.000
	6	1.000	0.999	0.973	0.920	0.825	0.527	0.227	0.058	0.007	0.002	0.000	0.000	0.000
	7	1.000	1.000	0.993	0.973	0.926	0.716	0.402	0.142	0.026	0.007	0.001	0.000	0.000
	8	1.000	1.000	0.999	0.993	0.974	0.858	0.598	0.284	0.074	0.027	0.007	0.000	0.000
	9	1.000	1.000	1.000	0.998	0.993	0.942	0.773	0.473	0.175	0.080	0.027	0.001	0.000
	10	1.000	1.000	1.000	1.000	0.998	0.981	0.895	0.671	0.340	0.190	0.082	0.003	0.000
	11	1.000	1.000	1.000	1.000	1.000	0.995	0.962	0.833	0.550	0.370	0.202	0.017	0.001
	12	1.000	1.000	1.000	1.000	1.000	0.999	0.989	0.935	0.754	0.595	0.402	0.068	0.007
	13	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.982	0.901	0.803	0.648	0.211	0.043
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.974	0.937	0.859	0.485	0.189
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.990	0.972	0.815	0.560
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

TABLE A.1 Cumulative binomial distribution (continued)

n	x	p												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
17	0	0.418	0.167	0.023	0.008	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.792	0.482	0.118	0.050	0.019	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.950	0.762	0.310	0.164	0.077	0.012	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.991	0.917	0.549	0.353	0.202	0.046	0.006	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.999	0.978	0.758	0.574	0.389	0.126	0.025	0.003	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.995	0.894	0.765	0.597	0.264	0.072	0.011	0.001	0.000	0.000	0.000	0.000
	6	1.000	0.999	0.962	0.893	0.775	0.448	0.166	0.035	0.003	0.001	0.000	0.000	0.000
	7	1.000	1.000	0.989	0.960	0.895	0.641	0.315	0.092	0.013	0.003	0.000	0.000	0.000
	8	1.000	1.000	0.997	0.988	0.960	0.801	0.500	0.199	0.040	0.012	0.003	0.000	0.000
	9	1.000	1.000	1.000	0.997	0.987	0.908	0.685	0.359	0.105	0.040	0.011	0.000	0.000
	10	1.000	1.000	1.000	0.999	0.997	0.965	0.834	0.552	0.225	0.107	0.038	0.001	0.000
	11	1.000	1.000	1.000	1.000	0.999	0.989	0.928	0.736	0.403	0.235	0.106	0.005	0.000
	12	1.000	1.000	1.000	1.000	1.000	0.997	0.975	0.874	0.611	0.426	0.242	0.022	0.001
	13	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.954	0.798	0.647	0.451	0.083	0.009
	14	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.988	0.923	0.836	0.690	0.238	0.050
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.981	0.950	0.882	0.518	0.208
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.992	0.977	0.833	0.582
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
18	0	0.397	0.150	0.018	0.006	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.774	0.450	0.099	0.039	0.014	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.942	0.734	0.271	0.135	0.060	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.989	0.902	0.501	0.306	0.165	0.033	0.004	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.998	0.972	0.716	0.519	0.333	0.094	0.015	0.001	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.994	0.867	0.717	0.534	0.209	0.048	0.006	0.000	0.000	0.000	0.000	0.000
	6	1.000	0.999	0.949	0.861	0.722	0.374	0.119	0.020	0.001	0.000	0.000	0.000	0.000
	7	1.000	1.000	0.984	0.943	0.859	0.563	0.240	0.058	0.006	0.001	0.000	0.000	0.000
	8	1.000	1.000	0.996	0.981	0.940	0.737	0.407	0.135	0.021	0.005	0.001	0.000	0.000
	9	1.000	1.000	0.999	0.995	0.979	0.865	0.593	0.263	0.060	0.019	0.004	0.000	0.000
	10	1.000	1.000	1.000	0.999	0.994	0.942	0.760	0.437	0.141	0.057	0.016	0.000	0.000
	11	1.000	1.000	1.000	1.000	0.999	0.980	0.881	0.626	0.278	0.139	0.051	0.001	0.000
	12	1.000	1.000	1.000	1.000	1.000	0.994	0.952	0.791	0.466	0.283	0.133	0.006	0.000
	13	1.000	1.000	1.000	1.000	1.000	0.999	0.985	0.906	0.667	0.481	0.284	0.028	0.002
	14	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.967	0.835	0.694	0.499	0.098	0.011
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.992	0.940	0.865	0.729	0.266	0.058
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.986	0.961	0.901	0.550	0.226
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.994	0.982	0.850	0.603
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	
19	0	0.377	0.135	0.014	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.755	0.420	0.083	0.031	0.010	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE A.1 Cumulative binomial distribution (continued)

n	x	p												
		0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95
19	2	0.933	0.705	0.237	0.111	0.046	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.987	0.885	0.455	0.263	0.133	0.023	0.002	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.998	0.965	0.673	0.465	0.282	0.070	0.010	0.001	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.991	0.837	0.668	0.474	0.163	0.032	0.003	0.000	0.000	0.000	0.000	0.000
	6	1.000	0.998	0.932	0.825	0.666	0.308	0.084	0.012	0.001	0.000	0.000	0.000	0.000
	7	1.000	1.000	0.977	0.923	0.818	0.488	0.180	0.035	0.003	0.000	0.000	0.000	0.000
	8	1.000	1.000	0.993	0.971	0.916	0.667	0.324	0.088	0.011	0.002	0.000	0.000	0.000
	9	1.000	1.000	0.998	0.991	0.967	0.814	0.500	0.186	0.033	0.009	0.002	0.000	0.000
	10	1.000	1.000	1.000	0.998	0.989	0.912	0.676	0.333	0.084	0.029	0.007	0.000	0.000
	11	1.000	1.000	1.000	1.000	0.997	0.965	0.820	0.512	0.182	0.077	0.023	0.000	0.000
	12	1.000	1.000	1.000	1.000	0.999	0.988	0.916	0.692	0.334	0.175	0.068	0.002	0.000
	13	1.000	1.000	1.000	1.000	1.000	0.997	0.968	0.837	0.526	0.332	0.163	0.009	0.000
	14	1.000	1.000	1.000	1.000	1.000	0.999	0.990	0.930	0.718	0.535	0.327	0.035	0.002
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.977	0.867	0.737	0.545	0.115	0.013
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.995	0.954	0.889	0.763	0.295	0.067
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.990	0.969	0.917	0.580	0.245
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.996	0.986	0.865	0.623
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	0	0.358	0.122	0.012	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	1	0.736	0.392	0.069	0.024	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.925	0.677	0.206	0.091	0.035	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.984	0.867	0.411	0.225	0.107	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.997	0.957	0.630	0.415	0.238	0.051	0.006	0.000	0.000	0.000	0.000	0.000	0.000
	5	1.000	0.989	0.804	0.617	0.416	0.126	0.021	0.002	0.000	0.000	0.000	0.000	0.000
	6	1.000	0.998	0.913	0.786	0.608	0.250	0.058	0.006	0.000	0.000	0.000	0.000	0.000
	7	1.000	1.000	0.968	0.898	0.772	0.416	0.132	0.021	0.001	0.000	0.000	0.000	0.000
	8	1.000	1.000	0.990	0.959	0.887	0.596	0.252	0.057	0.005	0.001	0.000	0.000	0.000
	9	1.000	1.000	0.997	0.986	0.952	0.755	0.412	0.128	0.017	0.004	0.001	0.000	0.000
	10	1.000	1.000	0.999	0.996	0.983	0.872	0.588	0.245	0.048	0.014	0.003	0.000	0.000
	11	1.000	1.000	1.000	0.999	0.995	0.943	0.748	0.404	0.113	0.041	0.010	0.000	0.000
	12	1.000	1.000	1.000	1.000	0.999	0.979	0.868	0.584	0.228	0.102	0.032	0.000	0.000
	13	1.000	1.000	1.000	1.000	1.000	0.994	0.942	0.750	0.392	0.214	0.087	0.002	0.000
	14	1.000	1.000	1.000	1.000	1.000	0.998	0.979	0.874	0.584	0.383	0.196	0.011	0.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.949	0.762	0.585	0.370	0.043	0.003
	16	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.984	0.893	0.775	0.589	0.133	0.016
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.965	0.909	0.794	0.323	0.075
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.992	0.976	0.931	0.608	0.264
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.988	0.878	0.642
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	

| Example - 04

- A large industrial firm allows a discount on any invoice that is paid within 30 days. Of all invoices, **10%** receive the discount. In a company audit, **12 invoices** are sampled at random. What is the probability that fewer than 4 of the 12 sampled invoices receive the discount?
- Let X represent the number of invoices in the sample that receive discounts. Then $X \sim \text{Bin}(12, 0.1)$. The probability that fewer than four invoices receive discounts is $P(X \leq 3)$. We consult **Table A.1** with $n = 12$, $p = 0.1$, and $x = 3$. We find that $P(X \leq 3) = 0.974$

| Example: Testing for Gender Bias in Promotions

- A group of women employees has claimed that female employees are less likely than male employees of similar qualifications to be promoted.
- Of 1000 employees, 50% are female.
- None of the 10 employees chosen for management training were female.
- The probability that no females are chosen is:

$$P(0) = \frac{10!}{0!10!} (0.50)^0 (0.50)^{10} = 0.001$$

- It is very unlikely (one chance in a thousand) that none of the 10 selected for management training would be female if the employees were chosen randomly.

| Check to See If Binomial Conditions Apply

Before using the binomial distribution, check that its three conditions apply:

- Binary data (success or failure)
- The same probability of success for each trial (denoted by p)
- Independent trials

| Example: Testing for Gender Bias in Promotions

Do the Binomial Conditions Apply?

- The data are binary (male, female).
- If employees are selected randomly, the probability of selecting a female on any given trial is 0.50.
- With random sampling of 10 employees from a large
- population, the outcome for one trial doesn't depend on
- the outcome of another trial.

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy