Statistics

Binomial Distribution

Page 212

4.

p = probability of "outcome of interest"

= probability of success

Flight arrive on time

$p = 0.75$

$n = 10$

$X \sim Bin(10, 0.75)$

(a) $P(10) = \binom{10}{10} (0.75)^{10} (0.25)^{0}$

$= 0.0563 = 5.6\ \%$

(b) $P(8) = \binom{10}{8} (0.75)^{8} (0.25)^{2}$

$= \frac{10!}{8!\,2!} (0.75)^{8} (0.25)^{2}$

$= 45 * 0.1001 * 0.0625$

$= 0.28156$

$= 28.16\ \%$

(c)

$$P(X \geq 8) = P(8) + P(9) + P(10)$$

OR from table

$$P(X \geq 8) = 1 - P(X \leq 7)$$
$$= 1 - 0.474$$
$$= 0.526 = 52.6\ \%$$

5.

$$p = 0.1$$
$$n = 12$$

(a) $$P(x) = \binom{n}{x} (p)^{x} (1-p)^{n-x}$$

$$P(3) = \binom{12}{3} (0.1)^{3} (0.9)^{9}$$

$$= 0.08523 = 8.5\ \%$$

(b) $P(X < 3) = P(X \leq 2) \Rightarrow$ from table

$$P(X \leq 2) = 0.889 = 89\ \%$$

(c) $$P(X = 0) = \binom{12}{0} (0.1)^{0} (0.9)^{12} = 0.2824$$

$$= 28.24\ \%$$

or from table $\Rightarrow$ 0.282 $\Rightarrow$ 28.2 %

8.

probability of success = probability of cost overruns

$= 0.2$

$p = 0.2$

$n = 20$

(a) $P(4) = \binom{20}{4} (0.2)^{4} (0.8)^{16}$

$= 0.218194 \simeq 21.8\%$

(b)

$P(X < 3) = P(X \leq 2) \rightarrow$ from table

$= 0.206 = 20.6\%$

(c) $P(0) = \binom{20}{0} (0.2)^{0} (0.8)^{20} = 0.0115$

from table $= 0.012 = 1.2\%$

14.

$P(\text{conform}) = 0.8$

$P(\text{downgrade}) = 0.15$

$P(\text{scrap}) = 0.05$

$n = 10$

if scrap is the outcome of interest

$\therefore \quad p = 0.05$

$n = 10$

$P(X \geq 1) = 1 - P(0)$

$= 1 - \binom{10}{0} (0.05)^{0} (0.95)^{10}$

$= 1 - 0.5987$

$= 0.40 = 40\%$

(b)

Not scrap is the outcome of interest

$p = 0.95$

$P(X \geq 8) = 1 - (P \leq 7)$

$= 1 - 0.012$ ← from table

$= 0.988 = 98.8\%$

(c)

either downgrade or scrap = Not conform

$\therefore p = 0.2$

$n = 10$

$P(x \geq 2) = 1 - P(x \leq 1)$

$= 1 - 0.376$

$= 0.624$

$= 62.4\ \%$

(d)

Conform or downgraded = Not scrap

$p = 0.95$

$P(9) = \binom{10}{9} (0.95)^{9} (0.05)^{1}$

$= 0.315 = 31.5\ \%$

We estimate p^4 with $\widehat{p}^4 = 0.93^4 = 0.7481$. We use Equation (3.10) to compute the uncertainty in $\widehat{p}^4$:

$$\begin{aligned}\sigma_{\widehat{p}^4} &\approx \left|\frac{d}{d\widehat{p}}\widehat{p}^4\right|\sigma_{\widehat{p}} \\ &= 4\widehat{p}^3\sigma_{\widehat{p}} \\ &= 4(0.93)^3(0.0255) \\ &= 0.082\end{aligned}$$

Exercises for Section 4.2

1. Let $X \sim \text{Bin}(7, 0.3)$. Find

a. $P(X = 1)$
b. $P(X = 2)$
c. $P(X < 1)$
d. $P(X > 4)$
e. μ_X
f. σ_X^2

2. Let $X \sim \text{Bin}(9, 0.4)$. Find

a. $P(X > 6)$
b. $P(X \geq 2)$
c. $P(2 \leq X < 5)$
d. $P(2 < X \leq 5)$
e. $P(X = 0)$
f. $P(X = 7)$
g. μ_X
h. σ_X^2

3. Find the following probabilities:

a. $P(X = 2)$ when $X \sim \text{Bin}(4, 0.6)$
b. $P(X > 2)$ when $X \sim \text{Bin}(8, 0.2)$
c. $P(X \leq 2)$ when $X \sim \text{Bin}(5, 0.4)$
d. $P(3 \leq X \leq 5)$ when $X \sim \text{Bin}(6, 0.7)$

4. At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

a. Find the probability that all 10 of the flights were on time.
b. Find the probability that exactly eight of the flights were on time.
c. Find the probability that eight or more of the flights were on time.

5. Of all the registered automobiles in a certain state, 10% violate the state emissions standard. Twelve automobiles are selected at random to undergo an emissions test.

a. Find the probability that exactly three of them violate the standard.
b. Find the probability that fewer than three of them violate the standard.
c. Find the probability that none of them violate the standard.

6. A fair die is rolled 8 times.

a. What is the probability that the die comes up 6 exactly twice?
b. What is the probability that the die comes up an odd number exactly five times?
c. Find the mean number of times a 6 comes up.
d. Find the mean number of times an odd number comes up.
e. Find the standard deviation of the number of times a 6 comes up.
f. Find the standard deviation of the number of times an odd number comes up.

7. Of all the weld failures in a certain assembly, 85% of them occur in the weld metal itself, and the remaining 15% occur in the base metal. A sample of 20 weld failures is examined.

a. What is the probability that exactly five of them are base metal failures?
b. What is the probability that fewer than four of them are base metal failures?
c. What is the probability that none of them are base metal failures?

d. Find the mean number of base metal failures.

e. Find the standard deviation of the number of base metal failures.

8. A general contracting firm experiences cost overruns on 20% of its contracts. In a company audit, 20 contracts are sampled at random.

a. What is the probability that exactly four of them experience cost overruns?

b. What is the probability that fewer than three of them experience cost overruns?

c. What is the probability that none of them experience cost overruns?

d. Find the mean number that experience cost overruns.

e. Find the standard deviation of the number that experience cost overruns.

9. Several million lottery tickets are sold, and 60% of the tickets are held by women. Five winning tickets will be drawn at random.

a. What is the probability that three or fewer of the winners will be women?

b. What is the probability that three of the winners will be of one gender and two of the winners will be of the other gender?

10. A quality engineer takes a random sample of 100 steel rods from a day's production, and finds that 92 of them meet specifications.

a. Estimate the proportion of that day's production that meets specifications, and find the uncertainty in the estimate.

b. Estimate the number of rods that must be sampled to reduce the uncertainty to 1%.

11. In a random sample of 100 parts ordered from vendor A, 12 were defective. In a random sample of 200 parts ordered from vendor B, 10 were defective.

a. Estimate the proportion of parts from vendor A that are defective, and find the uncertainty in the estimate.

b. Estimate the proportion of parts from vendor B that are defective, and find the uncertainty in the estimate.

c. Estimate the difference in the proportions, and find the uncertainty in the estimate.

12. Of the items manufactured by a certain process, 20% are defective. Of the defective items, 60% can be repaired.

a. Find the probability that a randomly chosen item is defective and cannot be repaired.

b. Find the probability that exactly 2 of 20 randomly chosen items are defective and cannot be repaired.

13. Of the bolts manufactured for a certain application, 90% meet the length specification and can be used immediately, 6% are too long and can be used after being cut, and 4% are too short and must be scrapped.

a. Find the probability that a randomly selected bolt can be used (either immediately or after being cut).

b. Find the probability that fewer than 9 out of a sample of 10 bolts can be used (either immediately or after being cut).

14. Gears produced by a grinding process are categorized either as conforming (suitable for their intended purpose), downgraded (unsuitable for the intended purpose but usable for another purpose), or scrap (not usable). Suppose that 80% of the gears produced are conforming, 15% are downgraded, and 5% are scrap. Ten gears are selected at random.

a. What is the probability that one or more is scrap?

b. What is the probability that eight or more are not scrap?

c. What is the probability that more than two are either downgraded or scrap?

d. What is the probability that exactly nine are either conforming or downgraded?

15. A commuter must pass through three traffic lights on her way to work. For each light, the probability that it is green when she arrives is 0.6. The lights are independent.

a. What is the probability that all three lights are green?

b. The commuter goes to work five days per week. Let X be the number of times out of the five days in a given week that all three lights are green. Assume the days are independent of one another. What is the distribution of X?

c. Find $P(X = 3)$.