Chapter 02

# Whole picture Example

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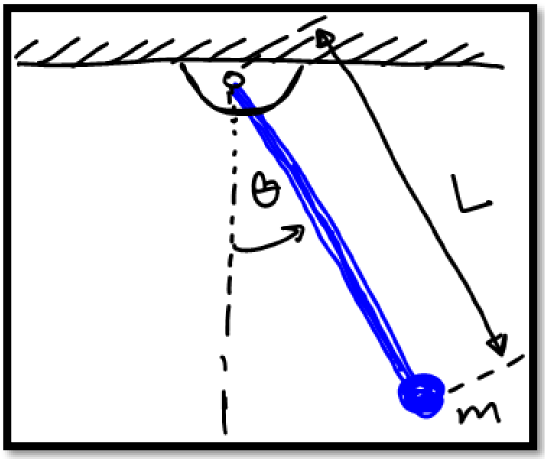
## ■ Picture 01

- Consider a simple pendulum having length $L$, mass $m$, and instantaneous angular displacement $\theta$, as shown in Figure.




where $\boldsymbol{L = 1\ m, m = 1\ kg}$, and $\boldsymbol{g = 9.8\ m/s^2}$

assume all the mass is concentrated at the end point and there a torque, $T_c$, applied at the pivot

use MATLAB to determine the time history of $\theta$ to a step input in $T_c$ of 1 N.m

$\sum T = I\alpha$

$I$ : moment of inertia = $mL^2$

Applying Newton 2nd law

$\therefore \quad T_c - mgL\sin\theta = I\ddot{\theta}$

$T_c - mgL\sin\theta = mL^2\ddot{\theta}$

$\ddot{\theta} + \frac{g}{L}\sin\theta = \frac{T_c}{mL^2}$

This is a nonlinear equation due to $\sin\theta$ term but in small $\theta$, $\sin\theta \approx \theta$

$$\therefore \quad \ddot{\theta} + \frac{g}{L}\theta = \frac{T_c}{mL^2}$$

Taking Laplace

$$\left(s^2 + \frac{g}{L}\right)\theta = \frac{T_c}{mL^2}$$

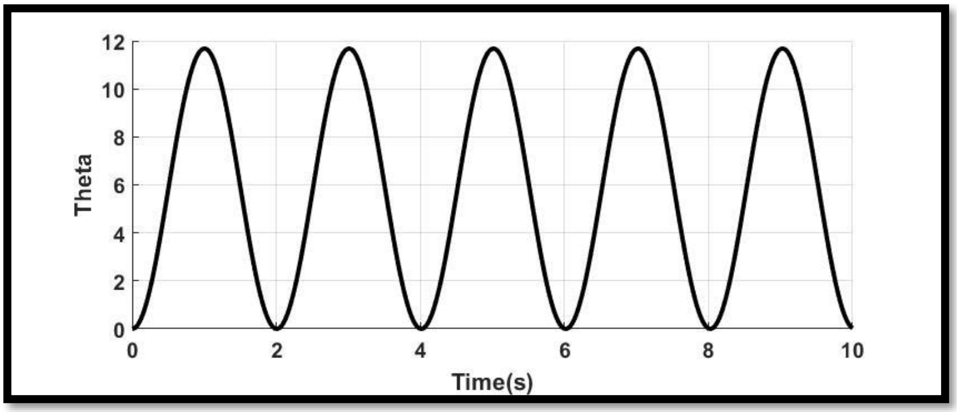
$$\frac{\theta}{T_c} = \frac{1/mL^2}{s^2 + \frac{g}{L}}$$

Substitute with values

$$\frac{\theta}{T_c} = \frac{1}{s^2 + 9.81}$$

**MATLAB code: `.m` file**

```
a = [1];
b = [1 0 9.81];
sys = tf(a,b);
t = 0:0.02:10;
y = step(sys,t);
plot(t,57.3*y);
```




As we saw, the resulting equation of motion:

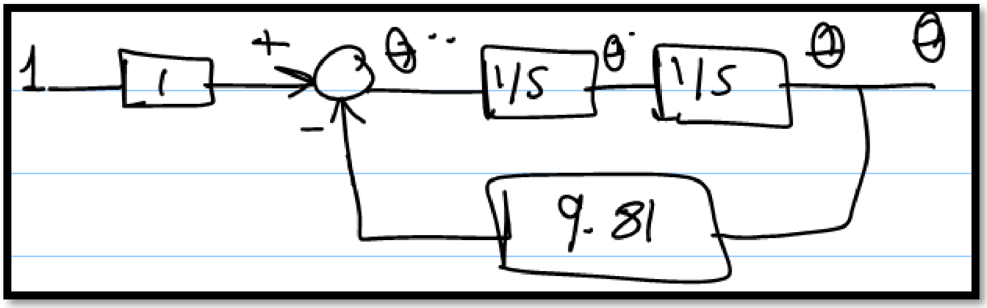
$$\ddot{\theta} + \frac{g}{L}\sin\theta = \frac{T_c}{mL^2}$$

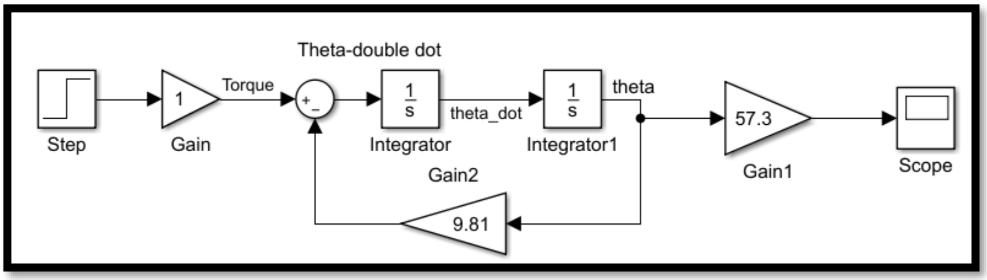
are more difficult to solve than linear ones

it is therefore useful to linearize model in order to gain access to linear analysis methods

$$\ddot{\theta} + \frac{g}{L}\theta = \frac{T_c}{mL^2}$$

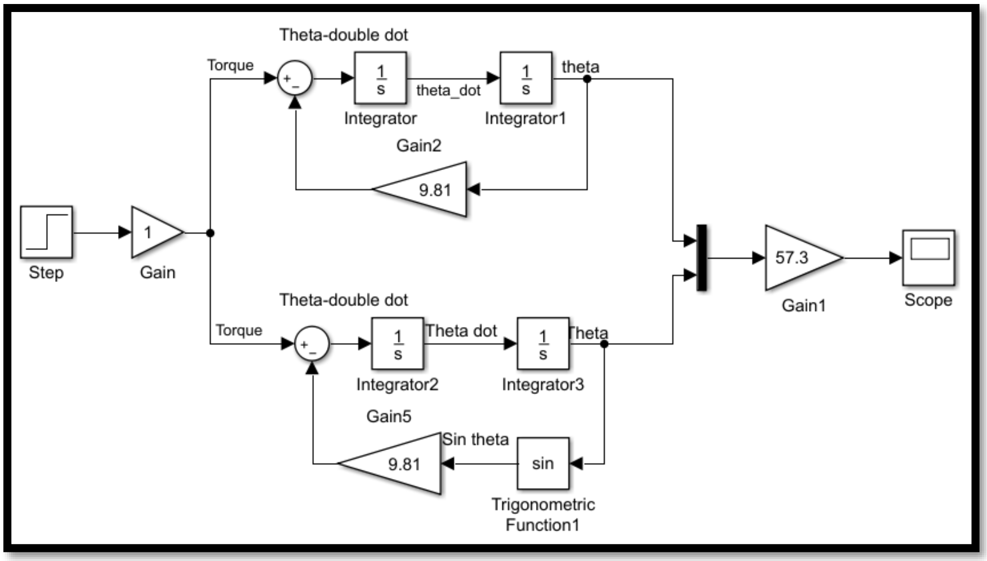
$$\ddot{\theta} = -9.81\,\theta + 1$$





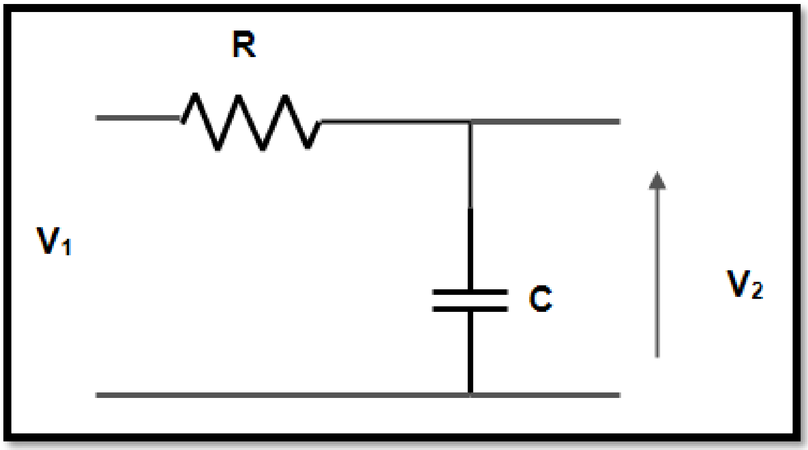

## Nonlinear model

$$\ddot{\theta} = -9.81 \sin \theta + 1$$



## ■ Picture 02:

Find the differential equation relating $\boldsymbol{v_1(t)}$ and $\boldsymbol{v_2(t)}$ for the $RC$ network shown in Figure below




$$v_1(t) = R\,i(t) + \frac{1}{c}\int i(t)\,dt \qquad \text{I}$$

$$v_2(t) = \frac{1}{c}\int i(t)\,dt \qquad \text{II}$$

Taking Laplace

$$V_1 = R\,I + \frac{I}{CS} = I(R + \frac{1}{CS})$$

$$V_2 = \frac{I}{CS}$$

Divide equation I by equation II, then

$$\frac{V_2}{V_1} = \frac{I/CS}{I/(R + \frac{1}{CS})} = \frac{1}{1 + RCS}$$

Assuming voltage source equal to 1 volt is applied the circuit, i.e. $v_1(t) = 1$

$\therefore \quad V_1(t) = 1/s$

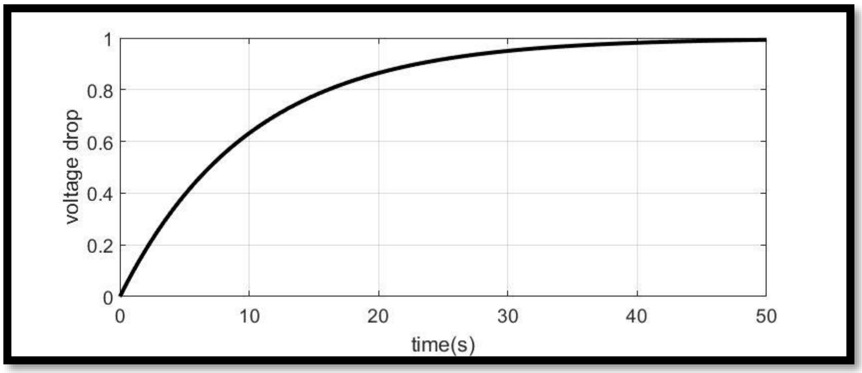
$\therefore \quad V_2 = \frac{1}{s(1+RCS)} = \frac{1}{s} - \frac{RC}{1+RCS} = \frac{1}{s} - \frac{1}{S+1/RC}$

$\therefore \quad v_2(t) = 1 - e^{-t/RC}$

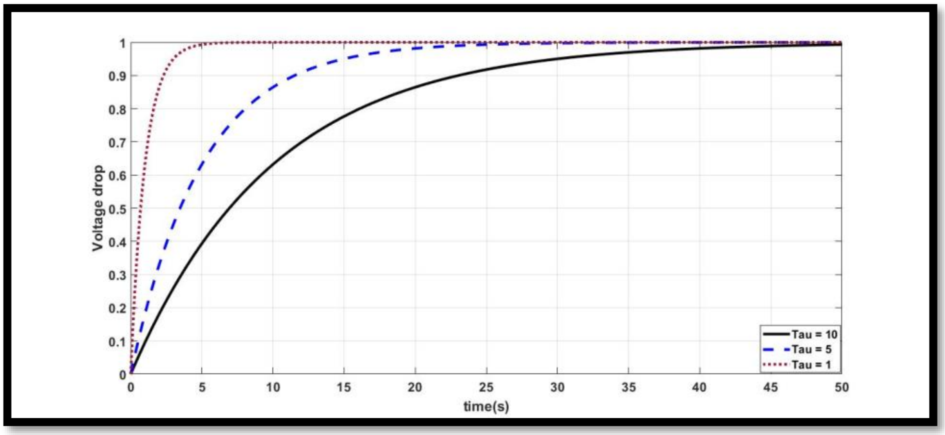
✍ Note:

$\tau$: **time constant** = $RC$

$R$ and $C$ are **physical parameters**, changing these values will change **time constant**, and finally this will have an impact on the **system response**

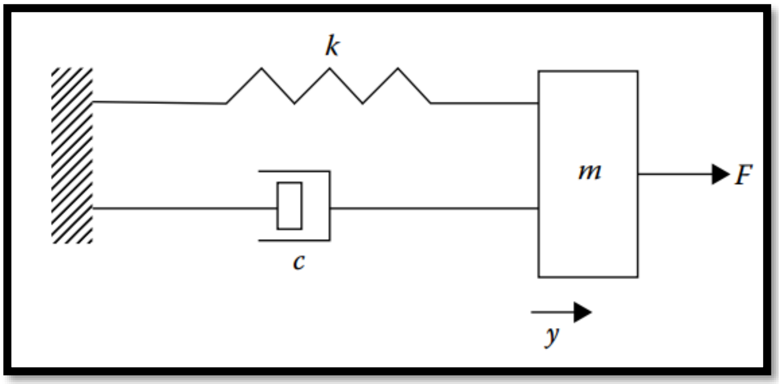


```
a = 1;
b1 = [10 1];
b2 = [5 1];
b3 = [1 1];
sys1 = tf(a,b1);
sys2 = tf(a,b2);
sys3 = tf(a,b3);
t = 0:0.1:50;
y1 = step(sys1,t);
y2 = step(sys2,t);
y3 = step(sys3,t);
plot(t,y1,t,y2,t,y3);
```



## ■ Picture 03:

Figure below shows a simple mechanical translational system with a **mass**, a **spring**, and a **dashpot**. A force ***F*** is applied to the system. Derive a **mathematical model** for the system.




As shown in figure, the net force on the mass is the applied force minus the forces exerted by the spring and the dashpot. Applying Newton's second law, we can write the system equation as

$$f(t) - c\dot{y}(t) - ky(t) = m\ddot{y}(t)$$

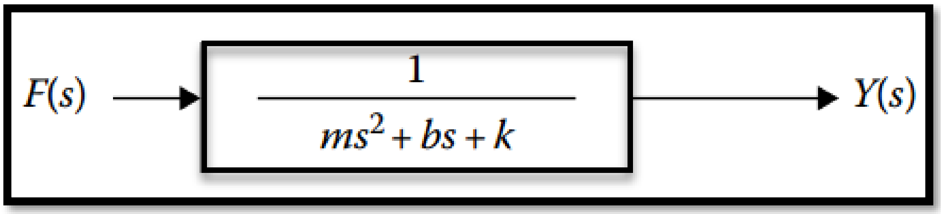
Taking Laplace

$$F(s) - csY(s) - kY(s) = ms^2Y(s)$$

$$Y(s)(ms^2 + cs + k) = F(s)$$

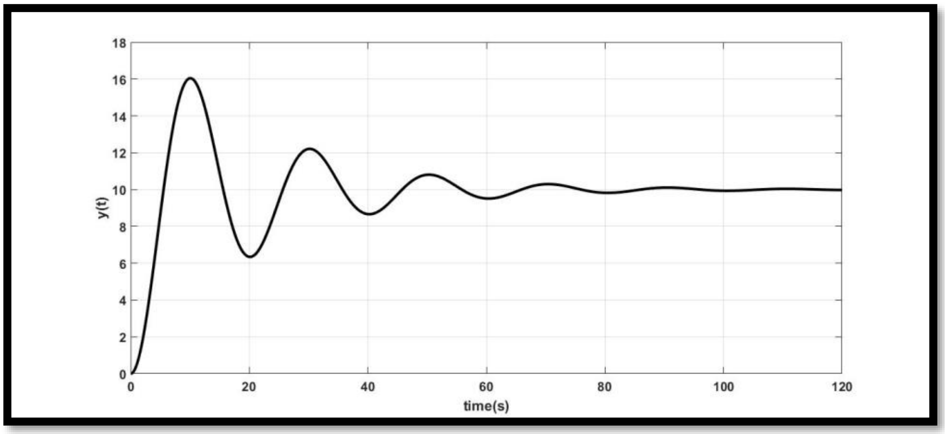
$\frac{Y(s)}{F(s)} = \frac{1}{ms^2+cs+k}$ ➔ second-order system

The **transfer function** in Equation above is represented by the **block diagram** shown in Figure below.

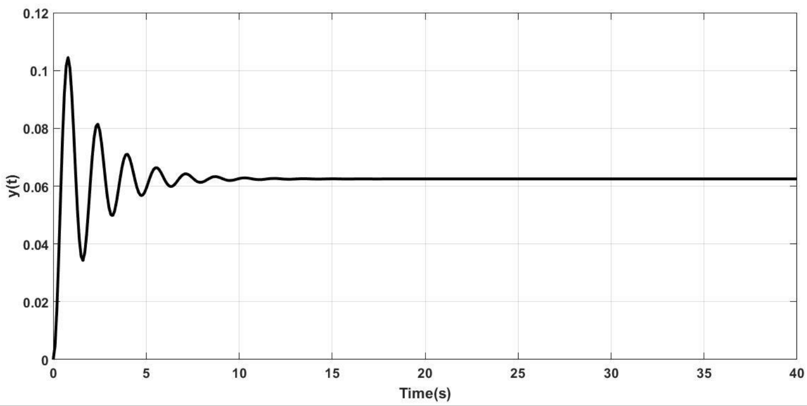



The **step response** of this mechanical system may be determined by simulating the following **MATLAB codes** and the response obtained is shown in Figure below.

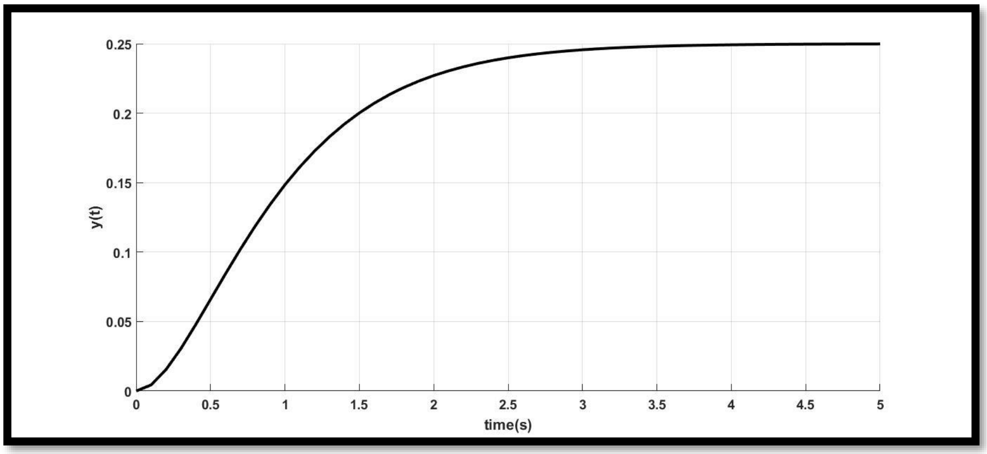
```
m=1.0; % kg
c=0.1; %
k=0.1; %
num = [1];
den=[m c k];
sys = tf(num,den);
t = 0:0.1:120;
y = step(sys,t);
plot(t,y);
```



```
m=1.0; % kg
c=1; %
k=16; %
num = [1];
den=[m c k];
sys = tf(num,den);
t = 0:0.1:120;
y = step(sys,t);
plot(t,y);
```




```
m=1.0; % kg
c=4; %
k=4; %
num = [1];
den=[m c k];
sys = tf(num,den);
t = 0:0.1:120;
y = step(sys,t);
plot(t,y);
```


0.25
0.2
0.15
0.1
0.05
0
y(t)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time(s)