

Probability

Bayes' Rule | Conditional Probability

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| Bayes' Rule

- If **A** and **B** are two events, we have seen that in most cases $P(A|B) \neq P(B|A)$.
- **Bayes' rule** provides a formula that allows us to calculate one of the conditional probabilities if we know the other one
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- Use $P(B|A)P(A)$ for $P(A \cap B)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

| Bayes' Rule

- When Bayes' rule is written, the expression **P(B)** in the **denominator** is usually replaced with a more complicated expression derived from the law of total probability
- $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

| Example - 01

- The **proportion** of people in a given community who have a certain **disease** is **0.005**. A test is available to diagnose the disease. If a person **has the disease**, the probability that the test will produce a **positive** signal is **0.99**. If a person **does not have the disease**, the probability that the **test will produce a positive signal** is **0.01**.

If a person tests positive, what is the probability that the person actually has the disease?

| Example - 02

- Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, **45%** have the **smallest** engine, **35%** have the **medium-sized** one, and **20%** have the **largest**. Of cars with the **smallest** engine, **10% fail** an emissions test within two years of purchase, while **12%** of those with the **medium** size and **15%** of those with the **largest** engine fail.

A record for a failed emissions test is chosen at random. **What is the probability that it is for a car with a small engine?**

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy