

Probability

Rule of Multiplication | Conditional Probability

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| Intro...

- Sometimes we know $P(A|B)$ and we wish to find $P(A \cap B)$.
- If A and B are two events with $P(B) \neq 0$, then

$$P(A \cap B) = P(B)P(A|B)$$

- If A and B are two events with $P(A) \neq 0$, then

$$P(A \cap B) = P(A)P(B|A)$$

- If $P(A) \neq 0$ and $P(B) \neq 0$, then both Equations hold

| Multiplication Rule

- If **A** and **B** are independent events, then

$$P(A \cap B) = P(A)P(B)$$

- This result can be extended to any number of events. If **A**, **B**, . . . , **Z** are independent events, then

$$P(A \cap B \cap C \dots \cap Z) = P(A)P(B)P(C) \dots P(Z)$$

| Example - 01

- A **vehicle** contains **two engines**, a **main** engine and a **backup**. The engine component **fails only if both engines fail**. The probability that the **main engine fails** is **0.05**, and the probability that the **backup engine fails** is **0.10**. Assume that the **main and backup** engines function **independently**.

What is the probability that the engine component fails?

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- A **vehicle** contains **two engines**, a **main** engine and a **backup**. The engine component **fails only if both engines fail**. The probability that the **main engine fails** is **0.05**, and the probability that the **backup engine fails** is **0.10**. Assume that the **main and backup** engines function **independently**.

What is the probability that the engine component fails?

Sol. -> 0.005

| Example - 02

- A **system** contains **two components, A and B**. **Both** components must function for the system to **work**. The probability that component **A fails is 0.08**, and the probability that component **B fails is 0.05**. Assume the **two components** function **independently**.

What is the probability that the system functions?

| Example - 02

- A **system** contains **two components, A and B. Both** components must function for the system to **work**. The probability that component **A fails is 0.08**, and the probability that component **B fails is 0.05**. Assume the **two components** function **independently**.

What is the probability that the system functions?

0.874

| Example - 03

- Of the **microprocessors** manufactured by a certain process, **20% are defective**. **Five** microprocessors are **chosen at random**. Assume they function **independently**.

What is the probability that they all work?

| Example - 03

- Of the **microprocessors** manufactured by a certain process, **20% are defective**. **Five** microprocessors are **chosen at random**. Assume they function **independently**.

What is the probability that they all work?

0.328

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy