Probability

Conditional Probability

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Conditional Probability

- A **sample space** contains all the possible outcomes of an experiment.
- Sometimes we obtain some **additional information** about an experiment that tells us that the **outcome comes** from a certain part of the sample space.
- In this case, the **probability of an event is based on** the outcomes in that part of the sample space.
- **A probability that is based on a part of a sample space** is called a conditional probability

Conditional Probability

- A population of **1000 aluminum rods**. For each rod, the **length** is classified as **too short**, **too long**, or **OK**, and the **diameter** is classified as **too thin**, **too thick**, or **OK**. These 1000 rods form a sample space in which **each rod is equally likely to be sampled**

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- **Of the 1000 rods**, 928 meet the diameter specification. Therefore, if a rod is sampled, P(diameter OK) = **928/1000** = 0.928.
- This probability is called the **unconditional probability**, since it is based on the entire sample space.

Conditional Probability

Length	Diameter: Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- Now **assume that a rod is sampled**, and its **length** is measured and found to **meet the specification**. What is the probability that the diameter also meets the specification?
- The **key** to computing this probability is to realize that **knowledge** that the **length meets the specification reduces the sample space** from which the rod is drawn
- Once we know that the **length specification is met**, we know that the rod will be **one of the 942 rods in the sample space**

Conditional Probability

- Of the 942 rods in this sample space, 900 of them meet the diameter specification
- probability that the rod meets the diameter specification is 900/942
- We say that the **conditional probability** that the rod meets the diameter specification given that it meets the length specification is equal to 900/942,
  - and we write **P(diameter OK | length OK)** = 900/942 = 0.955.

Example

- **P(diameter OK | length too long)?**

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

Example

- **P(diameter OK | length too long)?**

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- Of the 40 outcomes, 25 meet the diameter specification, therefore
- P(diameter OK | length too long) = 25/40 = 0.625

Note that

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

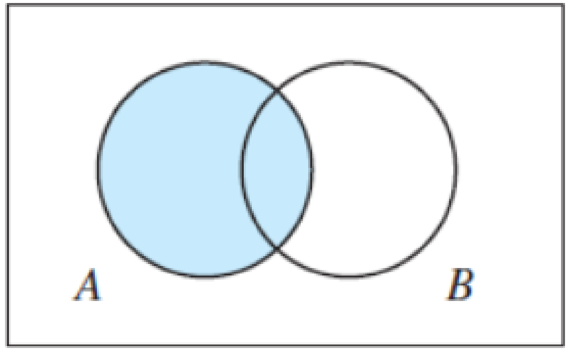
- If we divide both the numerator and denominator of this answer by the number of outcomes in the full sample space, which is 1000, we obtain P(diameter OK | length too long) = $\frac{25/1000}{\mathbf{40/1000}}$
- Now **40/1000** represents the **probability of satisfying the condition that the rod is too long**:
  - **P(length too long) = 40/1000**
- The quantity **25/1000** represents the **probability of satisfying both the condition that the rod is too long and that the diameter is OK**
  - **P(diameter OK and length too long) = 25/1000**

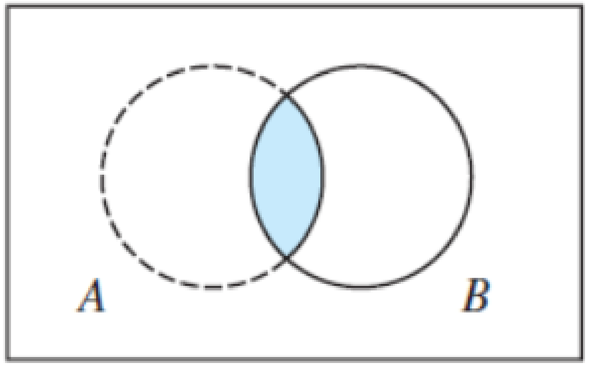
Note that

- P(diameter OK | length too long) = $\frac{\text{P(diameter OK and length too long)}}{\text{P(length too long)}}$
- Let A and B be events with $P(B) \neq 0$. The conditional probability of A given B is
- $\boldsymbol{P(A|B) = \frac{P(A \cap B)}{P(B)}}$

Venn Diagram

- Since **the event B is known to occur**, the **event B now becomes the sample space**.
- For the event A to occur, the outcome must be in the intersection $A \cap B$.
- The conditional probability **P(A|B)** is therefore illustrated by considering the intersection **A ∩ B in proportion to the entire event B**





Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the side, given that it has a flaw on top?**

Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the side, given that it has a flaw on top?**
- **P(flaw on top)=0.03**
- **P(flaw on side and flaw on top)=0.01**
- $\boldsymbol{P(Flaw\ on\ side\ |Flaw\ on\ top) = \frac{P(Flaw\ on\ side\ and\ Flaw\ on\ top)}{P(Flaw\ on\ top)}}$
- $\boldsymbol{P(Flaw\ on\ side\ |Flaw\ on\ top) = \frac{0.01}{0.03} = 0.33}$

Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the top, given that it has a flaw on the side?**

Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the top, given that it has a flaw on the side?**
- **P(flaw on side)=0.02**
- **P(flaw on side and flaw on top)=0.01**
- $\boldsymbol{P(Flaw\ on\ top\ |Flaw\ on\ side) = \frac{P(Flaw\ on\ top\ and\ Flaw\ on\ side)}{P(Flaw\ on\ side)}}$
- $\boldsymbol{P(Flaw\ on\ side\ |Flaw\ on\ top) = \frac{0.01}{0.02} = 0.5}$**, in Most cases** $\boldsymbol{P(A|B) \neq P(B|A)}$

Independent Events

- Sometimes the **knowledge that one event has occurred does not change the probability that another event occurs**.
- In this case the **conditional and unconditional** probabilities are the **same**, and the **events are said to be independent**

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Khan Academy