

Probability

Discrete Random Variable [Mean and Variance Examples]

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| Population Mean

- Let X be a discrete random variable with probability mass function $p(x) = P(X = x)$.

The mean of X is given by $\mu_X = \sum_x xP(X = x)$

where the sum is over all possible values of X .

| Population Variance

$$\sigma_X^2 = \sum_x (x - \mu_X)^2 P(X = x)$$

$$\sigma_X^2 = \sum_x x^2 P(X = x) - \mu_X^2$$

| Example - 01

- Consider **selecting at random** a student who is among the **15,000** registered for the current term at certain University. Let X = the number of courses for which the selected student is registered, and suppose that X has the following pmf:

x	1	2	3	4	5	6	7
$p(x)$	.01	.03	.13	.25	.39	.17	.02

- Find the expected value

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- Find the expected value
- 4.57

| Example - 02

- Just after birth, **each newborn** child is rated on a **scale** called the **Apgar** scale. The possible ratings are **0, 1, . . . , 10**, with the child's rating determined by **color**, **muscle** tone, **respiratory** effort, **heartbeat**, and **reflex** irritability (the **best possible score is 10**).
- Let X be the Apgar score of a randomly selected child born at a certain hospital **during the next year**, and suppose that the pmf of X is

x	0	1	2	3	4	5	6	7	8	9	10
$p(x)$	.002	.001	.002	.005	.02	.04	.18	.37	.25	.12	.01

- Find the mean value ?

| Example - 02

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- Find the mean value ?
- $E(x) = 7.15$

| Example - 03

- A resistor in a certain circuit is specified to have a resistance in the range $99\ \Omega - 101\ \Omega$. An engineer obtains **two resistors**. The probability that **both of them meet the specification** is **0.36**, the probability that exactly **one of them** meets the specification is **0.48**, and the probability that **neither of them** meets the specification is **0.16**. Let **X represent the number of resistors that meet the specification**.
- Find the probability mass function, and the mean, variance, and standard deviation of X.

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- Find the probability mass function, and the mean, variance, and standard deviation of X.
- $\mu_X = 1.2$ $\sigma_X^2 = 0.48$ $\sigma_X = 0.693$

| Prob - 01

- Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

- Find $P(X \leq 2)$.
- Find $P(X > 1)$.
- Find μ_X
- Find σ_X^2

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy