

First-Order Logic منطق الرتبة الأولى

Syntax · Semantics · Pragmatics

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Agenda

Three pillars of a language — then what we build with them

Part 01 · Syntax الصياغة

Structure البنية

Terms, predicates, functions, atoms, quantifiers

Part 02 · Semantics الدلالة

Meaning

Interpretation, denotation, logical consequence

Part 03 · Pragmatics التداولية

Use

The knowledge base — facts, rules, assumptions

Part 04 · Reasoning الاستدلال

Inference

Rules, unification, chaining, resolution

An argument nobody doubts

Every human is mortal.

Socrates is a human.

$\therefore$ Socrates is mortal.

In propositional logic we might write

p = Every human is mortal

q = Socrates is a human

r = Socrates is mortal

So the argument becomes

$p, q \models r ?$

The answer is no

$p, q \neq r$

Propositional logic sees only p, q, r — it does not know that:

human

appears in both p and q

Socrates

appears in both q and r

mortal

connects the rule to the conclusion

They *are* related in English. Propositional logic simply has **no machinery to represent that relationship** .

One legal assignment breaks it

$p = T$

Premise — true

$q = T$

Premise — true

$r = F$

Conclusion — false

This assignment is **perfectly legal** — p , q , r are independent atoms.

Both premises true, conclusion false → entailment **fails**. $p, q \not\models r$

We need to look inside the sentence

One atomic symbol

q



Structure exposed

Human (Socrates)

 Object: Socrates

 Property: Human

Mortal (Socrates)

“Every human is mortal” becomes

$\forall x \text{ (Human}(x) \rightarrow \text{Mortal}(x) \text{)}$

Now the entailment holds

```
rule   $\forall x \text{ (Human}(x) \rightarrow \text{Mortal}(x))$ 
```

```
fact  Human(Socrates)
```

```
 $\models$  Mortal(Socrates)
```

Before: $p, q \neq r$ — no shared structure

After: a **valid entailment** — the name and the predicates are shared

The nutshell

Propositional Logic

whole sentence $\rightarrow P$

Knows if it is true — not **what is inside**

Student (Ali)

Ali $\rightarrow$ object ·

Student $\rightarrow$ property

First-Order Logic

Human (Socrates)

Opens the sentence and represents its **structure**

Likes (Ali, Sara)

Ali, Sara $\rightarrow$ objects ·

Likes $\rightarrow$ relation

We need FOL because reasoning depends on **WHO**, **WHAT PROPERTY**, and **WHAT RELATION**.

Why do we need predicates مسند - محمول

They reveal structure that propositional symbols hide

Propositional logic

P = "Ali is a student"

Q = "Ahmed is a student"

R = "Sara is a student"

The common idea "student" is hidden.

Predicate logic

Student(Ali)

Student(Ahmed)

Student(Sara)

The shared structure is explicit.

Predicates let us **reuse the same concept for many objects.**

First-Order Logic

منطق الرتبة الأولى

Also called

Predicate Logic

Predicate Calculus

Both names because **predicates** are one of its central building blocks.

Why “first-order”?

The key is what the **variables** are allowed to represent

x, y, z

represent **individual objects** — a person, a car, a patient, a student, a number

$\forall x \text{ Student}(x) \rightarrow \text{HasID}(x)$

For **every object** x , if x is a student then x has an ID.

First Order = Quantify over Objects

$\forall x \text{ Human}(x) \rightarrow \text{Mortal}(x)$ — for every object x , if x is human then x is mortal.

What would “second-order” mean?

First-Order Logic

المتغير يمثل "شيئاً"

$\forall x \text{ Student}(x)$

x is an **object** — a person, a car, a number

Quantify over  **objects / individuals**

Second-Order Logic

المتغير قد يمثل "شيئاً" أو
"صفة/علاقة لهذا الشيء"

$\forall P \text{ P}(\text{Ali})$

P is a **property**, not an object

Quantify over objects **and** properties, relations, sets

First Order $\Rightarrow$ x, y, z represent individuals

FOL in a nutshell

Having predicates and functions does **not** make a logic second-order. The question is always: **what are we quantifying over?**

✓ Valid FOL

`Student (Ali)`

`Likes (Ali, Sara)`

`FatherOf (Ali)`

`$\forall x$  Student (x)  $\rightarrow$  Smart (x)`

`$\forall x$  (Student (x)  $\rightarrow$  HasID (x) )`

 Object: x

 Predicates: Student, HasID

 Quantifier: $\forall$.

 Connective: $\rightarrow$

FOL = Objects + Predicates + Functions + Quantifiers + Connectives

Second-order Logic

FOL quantifies over objects. Second-Order Logic can also quantify over properties and predicates.

X Not FOL

$\forall P - P$ ranging over arbitrary predicates

“Every property that Ali has is also possessed by Sara.”

In second-order logic, we could write something like:

$$\forall P (P(Ali) \rightarrow P(Sara))$$

Here P ranges over predicates such as:

- *Student*
- *Smart*
- *Tall*
- *Researcher*

That is why this is **not FOL**.

A robot looks at a dog

It sees

Pixels

It cannot say

$\text{Dog}(x)$

So it cannot reason

$\text{Mammal}(x)$

You cannot reason about what you cannot express.

An alien machine flashes symbols



Which combinations are allowed?

Syntax

What do they mean?

Semantics

How are they used?

Pragmatics

Without all three, symbols are **just noise**.

Syntax — structure, grammar, rules

Syntax defines which expressions are **well-formed**.
البنية النحوية أو قواعد الصياغة .

Lego. You cannot attach a wheel to empty space.

Wiring

Correct → the circuit works.

Wrong → sparks.

The cat my mother loves ✓

The my loves mother cat ✗

Dog (fido) ✓

Dog $\wedge$ fido ✗

Syntax tells us what is well-formed — **nothing about truth.**

كيف نكتب التعبير بطريقة صحيحة؟

Semantics — الدلالة

Syntax asks

Is it built correctly?

Semantics asks

ماذا يعني التعبير؟ وهل
هو صادق أم كاذب؟

What does it say about the world?

A map legend  — symbols are useless
until you know what they represent.

`Dog` → a **set of objects**

`OlderThan` → a **real relation**

Without interpretation, symbols **float in empty space**. → Part 2.

Pragmatics — use التداولية أو السياق الاستعمالي للمعنى

Blueprints are useless unless someone **builds** something.

How is the knowledge used to solve a problem, answer a query, make a decision, or support an action?

HighFever(Patient1)

HighFever(x) → NeedsAssessment(x)

- Pragmatics concerns how the system uses them, for example, to generate an alert:
Patient1 requires assessment.
- pragmatics is closely related to the **purpose and use of the knowledge base:**

Example

Student(Ali)

هل Student(Ali) مكتوبة وفق
قواعد اللغة المنطقية بشكل
صحيح؟ - بناء الجملة

Syntax

Syntax: How is it written?

ماذا تعني؟ - وهل هذه
العبارة صحيحة في العالم
الذي نمثله؟

Semantics

Semantics: What does it mean?

لماذا نستخدم هذه العبارة في
هذا الموقف؟ وما الذي نفهمه
منها حسب السياق؟

Pragmatics

Pragmatics: What does it mean in this context / why is it used?

What FOL lets us talk about

Its **ontological commitment** — what it says the world contains

Objects

`car1, fido`

Properties

`Vehicle(x)`

Relations

`In(p, cross)`

General rules

`$\forall x \dots$`

FOL lets us describe **structure in the world** — not just whole facts.