Logic Design

lecture 06 ⇝ Binary logic
⇓
Boolean Algebra

Boolean algebra ⇒ Deals with variables that takes on two discrete values and operations that assume logical meaning

true — false

Yes — No

1 , 0

So variable A in binary logic may be 1 or Zero.

Logical operation ⇒ AND — OR — Not

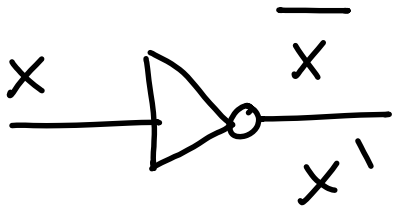
AND $X \cdot Y \Rightarrow XY \Rightarrow$ X anded Y

OR $X + Y \Rightarrow$ X ORed Y

NOT $\overline{X} \Rightarrow X' \Rightarrow$ X Complement

NOT

- Complement ⇒ Negate ⇒ Inverter
- Single input
- $\overline{X} \Rightarrow X'$



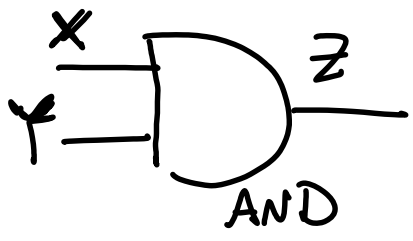
X	$\overline{X}$
0	1
1	0

⇒ Truth table

AND

- operation to check if two conditions are met
- two inputs or more
- outputs 1 if and only if both input are 1

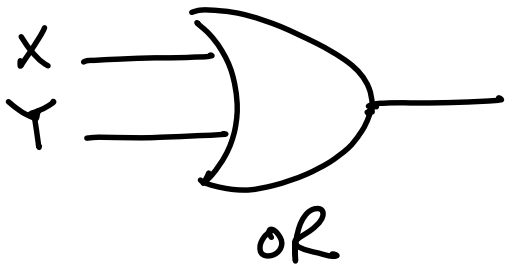
X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1



- $X \cdot Y \Rightarrow XY \Rightarrow X$ anded Y

OR

- operation to check if at least one condition is met
- may be two inputs or more
- outputs 1 if any input is 1
- $X + Y$



X	Y	OR
0	0	0
0	1	1
1	0	1
1	1	1

To conclude

X	Y	AND X·Y	OR X+Y	NOT X	NOT Y
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	1
1	1	1	1	0	0

Logic Gates

Circuits that operate on one or more input signals to produce an output signals

- AND
- OR
- NOT

Electrical Signals

- Voltage
- Current

Range of electrical signals

TTL

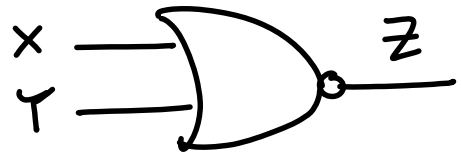
+ + 7400

7402

74 ——→

Other two-input gates

NoR ⇒



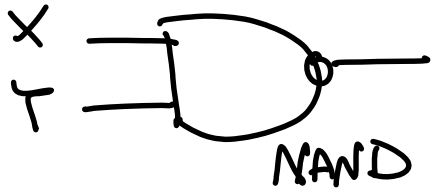
X	Y	X+Y	NoR $\overline{X+Y}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

$\Rightarrow Z = \overline{X+Y}$

Existence of 1 ⟹ 0 output

NAND

X	Y	AND X·Y	NAND $\overline{XY}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0



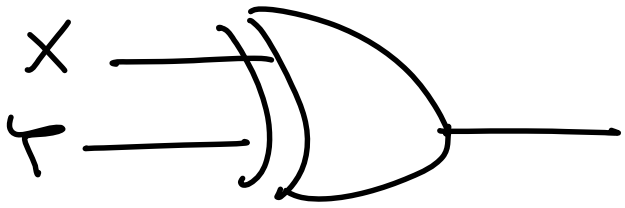
$Z = \overline{XY}$

Existence of 0 lead to 1

XOR

X	Y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive OR

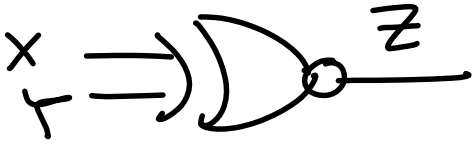


XOR

$$Z = X \oplus Y$$

if No. of 1 in input is odd, the output is 1

XNOR

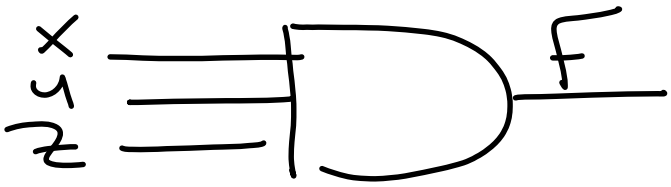


$$Z = \overline{X \oplus Y}$$

$$Z = X \odot Y$$

X	Y	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

Gates with more than 2 inputs



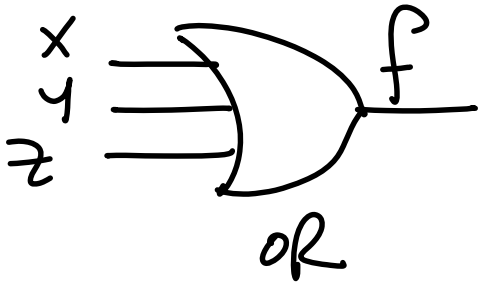
$f = xyz$

$= x \cdot y \cdot z$

x	y	z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Ex Truth table for three input OR

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



$f = x + y + z$

Boolean Algebra

$\overline{0} = 1$ || $A + 1 = 1$ || $A + \overline{A} = 1$

$\overline{1} = 0$ || $A + A = A$ || $A + 0 = A$

- $\overline{\overline{A}} = A$
- $A \cdot 0 = 0$
- $A \cdot 1 = A$
- $A \cdot A = A$
- $A \cdot \overline{A} = 0$
- $A(B + C) = AB + AC$

- $A + AB = A(1+B) = A(1) = A$
- $A \oplus \overline{A} = 1$
- $\overline{A+B} = \overline{A}\,\overline{B}$
- $A + BC = (A+B)(A+C)$

⇓ prove

A	B	C	BC	A+BC	A+B	A+C	(A+B)(A+C)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Ex Sketch Truth table, logic diagram

$F = x + \bar{y}z$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$F = 0 + \bar{0} \cdot 0$

$= 0 + 1 \cdot 0$

$= 0 + 0$

$= 0$

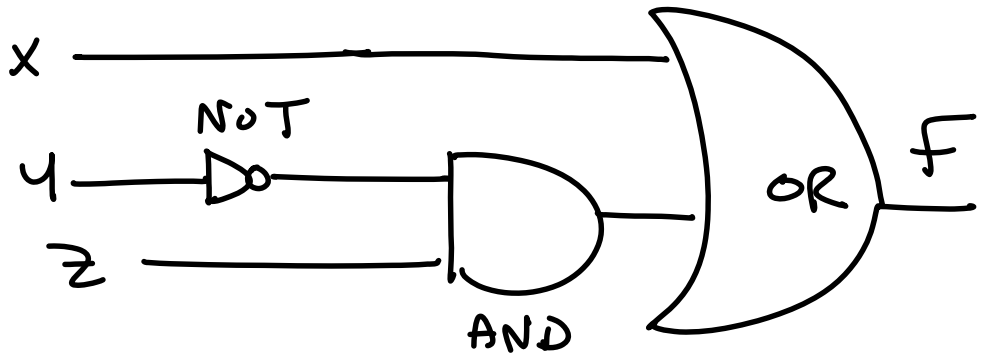
0 0 1

x y z

$F = x + \bar{y}z$

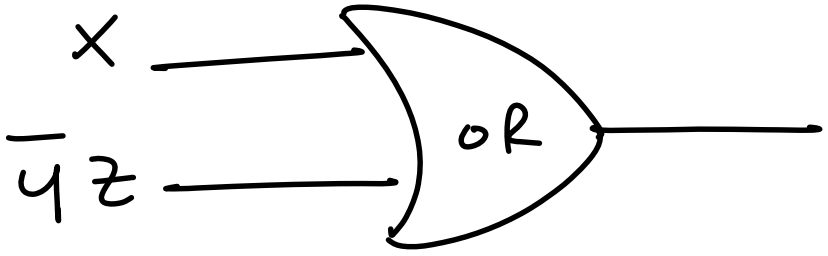
$= 0 + \bar{0} \cdot 1$

$= 0 + 1 \cdot 1 = 0 + 1 = 1$



NOTE

- this function is an OR



- Results in 1 if one side is 1

if x is 1 ⇒ output is 1

this is an AND gate

↓ continue