Agenda

- Product
- Sum
- minterm
- maxterm
- Sum_of_product
- product_of_Sum

Ex

Design a majority circuit that produce 1 if more half the inputs are all 1. Design for 3-inputs

Sol.

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\Rightarrow f = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

$$f = \bar{x}yz + x\bar{y}z + xy(z+\bar{z})$$

$$= \bar{x}yz + x\bar{y}z + xy$$

$$= z(x \oplus y) + xy$$

Canonical forms

product: the AND of one or more literals is called a product or an implicit, like $\bar{A}B$, AC, $B\bar{C}$ are all implicants for a function of three variables

minterm: is a product involving all of the inputs to the function

Ex $A\bar{B}\bar{C}$ is a minterm for a function of three variable A, B, C

$\overline{A}B$ is a product, Not a minterm because it does not involve $\underline{\underline{C}}$

Sum: The OR of one or more literals is called a sum. like $A+B$, $\overline{A}+B$, $A+B+\overline{C}$

maxterm: is a Sum involving all of the inputs to the function

Ex $A+\overline{B}+C$ is a maxterm for a function of the three Variables A, B, and C

		Minterm		Maxterm	
x	y	term	designation	term	designation
0	0	$\overline{x}\,\overline{y}$	m_0	$x+y$	M_0
0	1	$\overline{x}\,y$	m_1	$x+\overline{y}$	M_1
1	0	$x\,\overline{y}$	m_2	$\overline{x}+y$	M_2
1	1	$x\,y$	m_3	$\overline{x}+\overline{y}$	M_3

For three variables

x	y	z	term		term	
0	0	0	$\bar{x}\bar{y}\bar{z}$	m_0	$x+y+z$	M_0
0	0	1	$\bar{x}\bar{y}z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}y\bar{z}$	m_2	$x+\bar{y}+z$	M_2
0	1	1	$\bar{x}yz$	m_3	$x+\bar{y}+\bar{z}$	M_3
1	0	0	$x\bar{y}\bar{z}$	m_4	$\bar{x}+y+z$	M_4
1	0	1	$x\bar{y}z$	m_5	$\bar{x}+y+\bar{z}$	M_5
1	1	0	$xy\bar{z}$	m_6	$\bar{x}+\bar{y}+z$	M_6
1	1	1	xyz	m_7	$\bar{x}+\bar{y}+\bar{z}$	M_7

Sum-of-product SOP

We can write a boolean function for any truth table by summing each of the minterms for which the output, Y, is true

Ex

A	B	Y	minterm
0	0	0	$\bar{A}\bar{B}$
0	1	1	$\bar{A}B$
1	0	0	$A\bar{B}$
1	1	0	AB

minterm for which Y is equal to 1

$\therefore Y = \bar{A}B$

Ex

A	B	Y	minterm
0	0	0	$\bar{A}\bar{B}$
0	1	1	$\bar{A}B$
1	0	0	$A\bar{B}$
1	1	1	AB

$Y = \bar{A}B + AB = B$

Ex

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$$

Ex

Express the output of 3-input majority gate as SOP term.

Sol

A	B	C	F	minterm	Designation
0	0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0
0	0	1	0	$\bar{A}\bar{B}C$	m_1
0	1	0	0	$\bar{A}B\bar{C}$	m_2
0	1	1	1	$\bar{A}BC$	m_3
1	0	0	0	$A\bar{B}\bar{C}$	m_4
1	0	1	1	$A\bar{B}C$	m_5
1	1	0	1	$AB\bar{C}$	m_6
1	1	1	1	ABC	m_7

$F = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$F = m_3 + m_5 + m_6 + m_7$

$F = \sum(3, 5, 6, 7)$

think in it as

011 $\rightarrow$ $\bar{A}BC$

101

$A\bar{B}C$

<u>Ex</u> find the sum-of-product, and product-of-sum of the truth table shown below

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

$Y = A\bar{B} + AB$

$Y = m_2 + m_3$

$Y = \sum(2, 3)$

$$Y \text{ as } SOP = \sum(2,3)$$

$$Y = (A+B)(A+\bar{B}) \Rightarrow POS$$

$$Y = A + A\bar{B} + AB$$

$$= A(1 + B + \bar{B})$$

$$Y = A$$

$$Y = \pi(M_0, M_1)$$

$$= \pi(0,1)$$

Ex Find the SOP of the truth table shown below

x	y	z	f_1	f_2
0	0	0	0	1
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$\Rightarrow f_1 = \sum(1,4,7)$

$\Rightarrow f_2 = \sum(0,3,5,6,7)$

$$\therefore\ f_1(x,y,z) = \sum(\underset{001}{1}, \underset{100}{4}, \underset{111}{7})$$

$$f_1 = \bar{x}\bar{y}z + x\bar{y}\bar{z} + xyz$$

$$\therefore\ f_2(x,y,z) = \sum(0,3,5,6,7)$$

$$f_2 = \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

product_of_Sum : POS

we can write a boolean equation for any circuit directly from truth table as the AND of each of the maxterms which the output is false

Ex

A	B	C	Y	
0	0	0	0	M_0
0	0	1	1	
0	1	0	1	
0	1	1	1	
1	0	0	0	M_4
1	0	1	0	M_5
1	1	0	1	
1	1	1	0	M_7

$$Y = M_0 M_4 M_5 M_7$$

$$= \pi(0, 4, 5, 7)$$

000 100 101 111

$$Y = (A+B+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})$$
$$(\bar{A}+\bar{B}+\bar{C})$$

Ex

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Y as SoP

$$= \sum(1,2,3)$$
$$= \bar{A}B + A\bar{B} + AB$$
$$= \bar{A}B + A(\bar{B}+B)$$
$$= \bar{A}B + A$$
$$= (A+\bar{A})(A+B)$$
$$= A+B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Y as PoS

$$= \prod(0)$$
$$= A+B$$

Ex Express the truth table below in product-of-sum and sum-of-product, prove that, they are equivalent

A	B	f
0	0	0
0	1	1
1	0	0
1	1	1

$$f = \sum (1,3)$$
$$= \bar{A}B + AB$$
$$= B$$

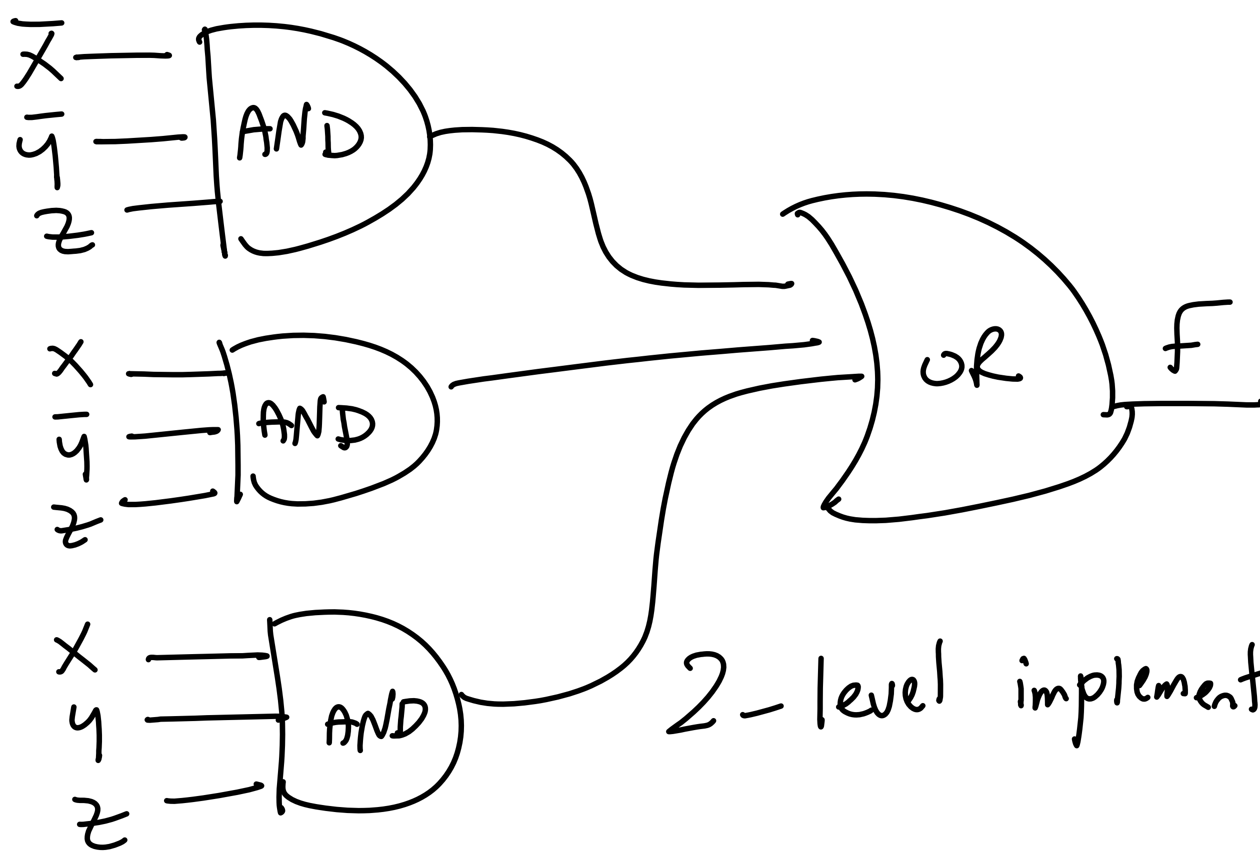
$$f = \prod (0,2) = (A+B)(\bar{A}+B)$$
$$= AB + \bar{A}B + B$$
$$= B(A + \bar{A} + 1)$$
$$= B$$

Ex implement the truth table below

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$\Rightarrow f = \sum(1,5,7)$

$$f = \bar{x}\bar{y}z + x\bar{y}z + xyz$$



2-level implementation

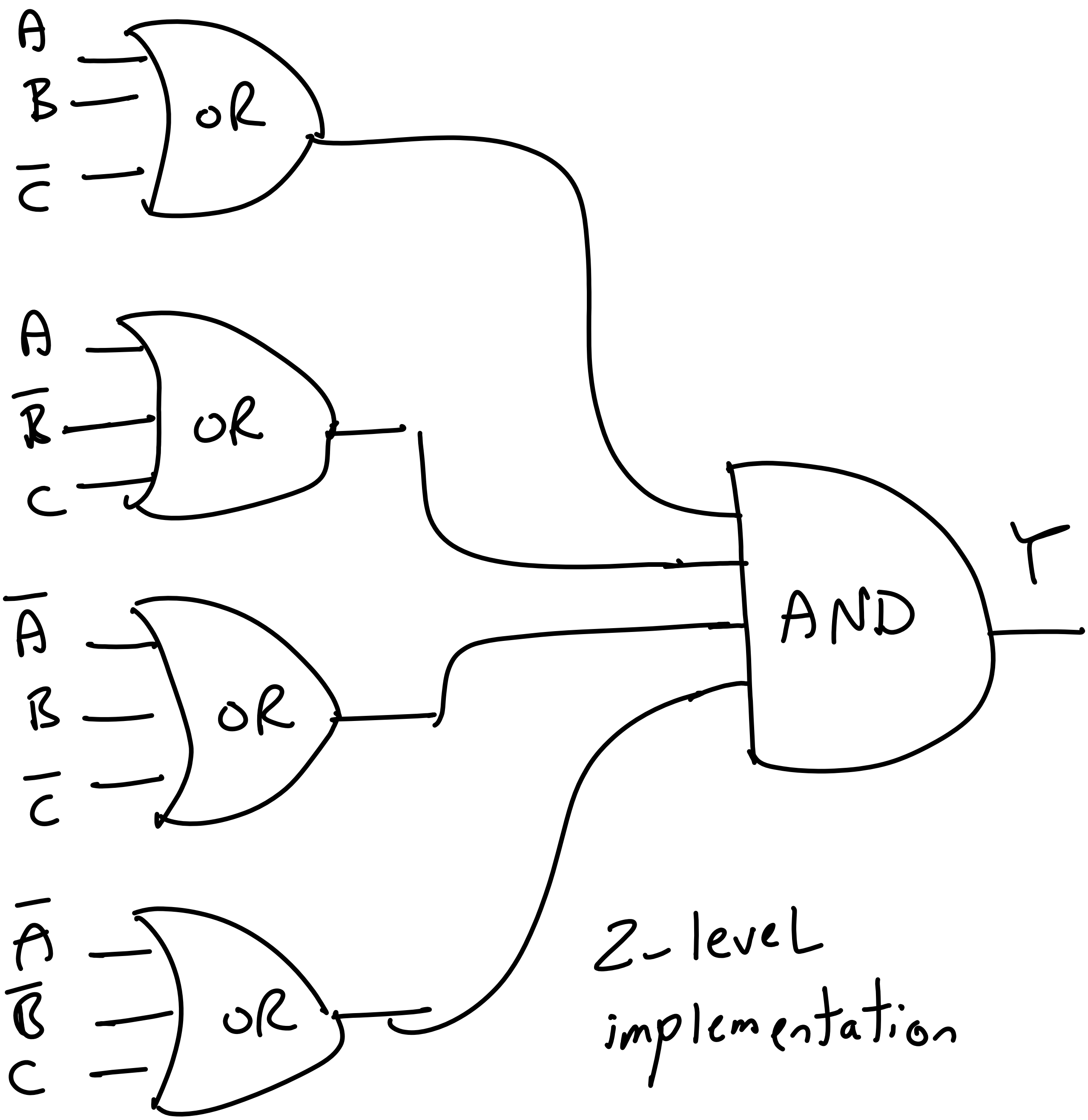
Ex write down the product_of_Sum form of the truth table shown below, implement this function.

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\Rightarrow Y = \pi(1, 2, 5, 6)$

001 010 101 110

$$Y = (A+B+\overline{C})(A+\overline{B}+C)(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+C)$$



2-level implementation

Ex write down SOP, and POS of the truth table shown below, implement them

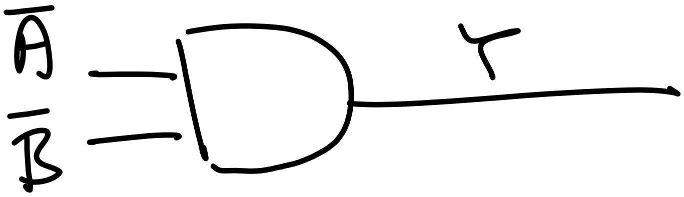
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

$\Rightarrow$

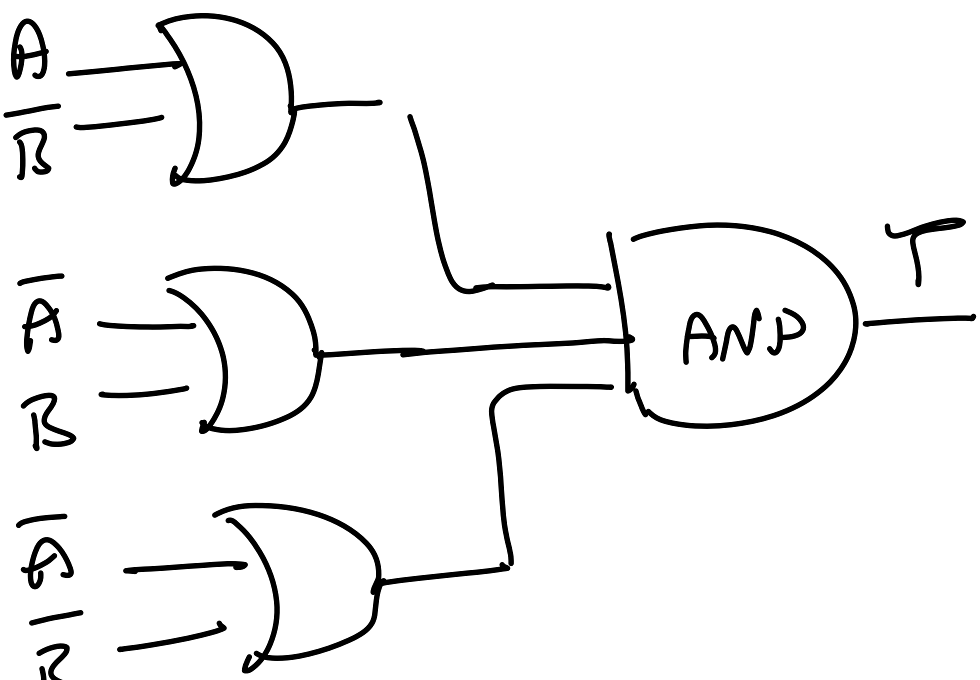
$$Y = \sum(0) = \overline{A}\,\overline{B}$$

$$Y = \prod(1,2,3)$$

$T \Rightarrow POS = (A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$



SOP ⇒ <u>1 Gate</u>



<u>4 Gate</u>

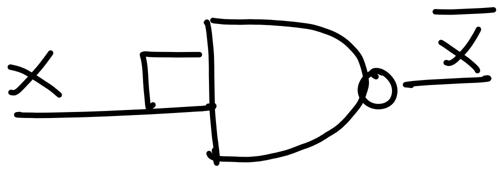
<u>unfortunately</u>, SOP, and pos does not necessarily produce the simplest equation, later we will come to that point

$\Downarrow$

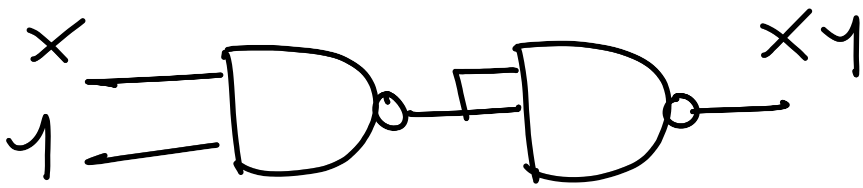
How to implement using fewer gates

NAND – NOR as universal Gate

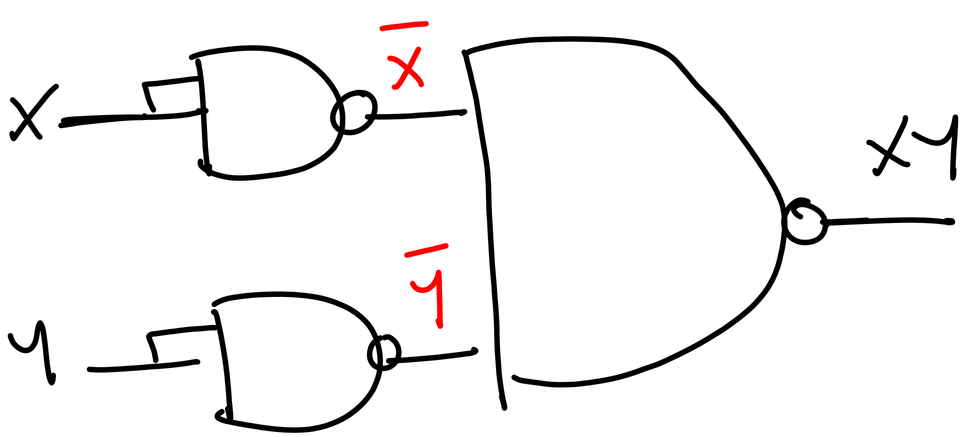
NOT using NAND



AND using NAND



OR using NAND



NOR as universal Gate $\Rightarrow$ Report

Ex $F = x + \bar{y}z$, is it in canonical form?

Sol No, but we can convert

mathematically

by truth table

$$F = x(y+\bar{y})(z+\bar{z}) + (x+\bar{x})\bar{y}z$$

$$= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z}$$

$$+ x\bar{y}z + \bar{x}\bar{y}z$$

$$= \sum(1,4,5,6,7)$$

by truth table $\qquad x + \bar{y} z$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$f = \sum(1, 4, 5, 6, 7)$

Ex Find the product_of_sum of

$F = xy + \bar{x} z$

(1) mathematically

$$f = (xy + \bar{x})(xy + z)$$

$$= (x + \bar{x})(\bar{x} + y)\,(x + z)(y + z)$$

$$= (\bar{x} + y)(x + z)(y + z)$$

$$= (\bar{x} + y + z\bar{z})(x + z + y\bar{y})(y + z + x\bar{x})$$

$$f = (\bar{x}+y+z)(\bar{x}+y+\bar{z})$$
$$(x+y+z)(x+\bar{y}+z)$$
$$(x+y+z)(\bar{x}+y+z)$$

$$f = \pi(4, 5, 0, 2, 0, 4)$$

$$f = \pi(0, 2, 4, 5)$$

② by truth table $\Rightarrow f = xy + \bar{x}z$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$f = \pi(0, 2, 4, 5)$$

Conversion between
canonical forms

$$f(x,y,z) = \sum(1,4,5,6,7)$$

$$\Downarrow$$

$$f(x,y,z) = \pi(0,2,3)$$

Ex Derive the truth table, and obtain sum of product and product of sum of

$$f = \overline{x} + yz$$

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$$f = \sum(0,1,2,3,7)$$

$$f = \pi(4,5,6)$$