Karnaugh map

Complexity of digital circuits directly related to the complexity of the algebraic expression we use to build the circuit.

Truth table may lead to different implementations, which one to use?

optimization techniques of algebraic expressions need more systematic (algorithmic way)

Karnaugh

Quine - McCluskey

Minimization of logic function ⟹ we have chips with million of gates

- why care about minimization of

a function?

- what do a few gates matter

- what is the criterion for minimization should we minimize
  1. Number of variables?
  2. Number of minterms?
  3. Number of logic operations?
  4. Number of wires?

For implementation

minimize number of gates

Ex

$$F = AB + A\overline{B}$$
$$= A(B+\overline{B}) = A$$

If we can put these two terms next to each other in square, and try to minimize it based on their adjacency

A	B	f	
0	0	0	m_0
0	1	0	m_1
1	0	1	m_2
1	1	1	m_3

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

→ Variable

A \ B	0	1
0		
1	1	1

constant →

$$f = A$$

Karnaugh map with two variable

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

Karnaugh map with three variables

X	Y	Z	F	
0	0	0		m_0
0	0	1		m_1
0	1	0		m_2
0	1	1		m_3
1	0	0		m_4
1	0	1		m_5
1	1	0		m_6
1	1	1		m_7

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$xy \backslash z$	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

Ex

minimize $F(x,y,z) = \sum(2,3,4,5)$ using Karnaugh map

$x \backslash yz$	00	01	11	10
0			1	1
1	1	1		

$$F = \bar{x}y + x\bar{y}$$

Ex

$$f(A,B) = AB$$

B=1

$A \backslash B$	0	1
0		
1		1

A=1

$f = AB$

A \ B	0	1
0		
1		1

Ex Minimize $F(x,y,z) = \sum(3,4,6,7)$ using Karnaugh map

x \ yz	00	01	11	10
0			1	
1	1		1	1

$F = x\bar{z} + yz$

Karnaugh map with 4 variables

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Ex

minimize

$$F(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$$

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

$$F = \bar{y} + \bar{w}\bar{z} + x\bar{z}$$

Ex minimize $Y = A + AC + BC$

A \ BC	00	01	11	10
0			1	
1	1	1	1	1

$$Y = A + BC$$

<u>Ex</u>

Simplify $F(A,B,C,D) = \sum(0,4,5,7,8,12,13,15)$

AB \ CD	00	01	11	10
00	1			
01	1	1	1	
11	1	1	1	
10	1			

$F = \bar{C}\bar{D} + BD + B\bar{C}$

AB \ CD	00	01	11	10
00	1			
01	1	1	1	
11	1	1	1	
10	1			

$F = \bar{C}\bar{D} + BD$

Ex Simplify F in truth table shown below

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

A \ BC	00	01	11	10
0	1			
1	1			1

start from here

$$F = \bar{B}\bar{C} + A\bar{C}$$

Ex ⇒ find F in product_of_sum minimized form for the truth table shown in example above

A \ BC	00	01	11	10
0		0	0	0
1		0	0	

$$\overline{f} = C + \overline{A} B$$

$\downarrow$

$$f = \overline{C} (A + B)$$

Note

to get the function in minimized product_of_ sum $\longrightarrow$ minimize 0 in karnaugh map, then $\bar{f} =$ ______, implement this function

Ex Simplify $F(x, y, z, t) = \sum (0, 1, 2, 5, 8, 9, 10)$ in SOP, POS

$\Downarrow$

xy \ zt	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1

pos

$\hookrightarrow F = \bar{y}\bar{z} + \bar{y}\bar{t} + \bar{x}\bar{z}t$

$\bar{F} = y\bar{t} + zt$

$+ xy$

xy \ zt	00	01	11	10
00			0	
01	0		0	0
11	0	0	0	0
10			0	

$f = (\bar{y} + t)(\bar{z} + \bar{t})$

$(\bar{x} + \bar{y})$

Ex

$$F(x,y,z) = \Pi(0,2,5,7)$$

$x \backslash yz$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

SOP

$\hookrightarrow$ $F = x\bar{z} + \bar{x}z$

POS

$\hookrightarrow$ $\bar{f} = \bar{x}\bar{z} + xz$

$$f = (x+z)(\bar{x}+\bar{z})$$

Ex Simplify the map below

Start from here

$A \backslash BC$	00	01	11	10
0	1		1	1
1	1			1

$$F = \bar{A}B + \bar{C}$$

Ex

Design a digital circuit that can convert from BCD to Gray code

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	0	1	1	1
0	1	1	0	0	1	0	1
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	0	1	1	1	0	1

↓ 10

↓ 16 $\Longrightarrow$ Do NoT care

W

AB \ CD	00	01	11	10
00				
01				
11	X	X	X	X
10	1	1	X	X

$W = A$

X

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	X	X	X	X
10	1	1	X	X

$X = B + A$

Y

AB \ CD	00	01	11	10
00			1	1
01	1	1		
11	X	X	X	X
10			X	X

$$Y = B\bar{C} + \bar{B}C$$

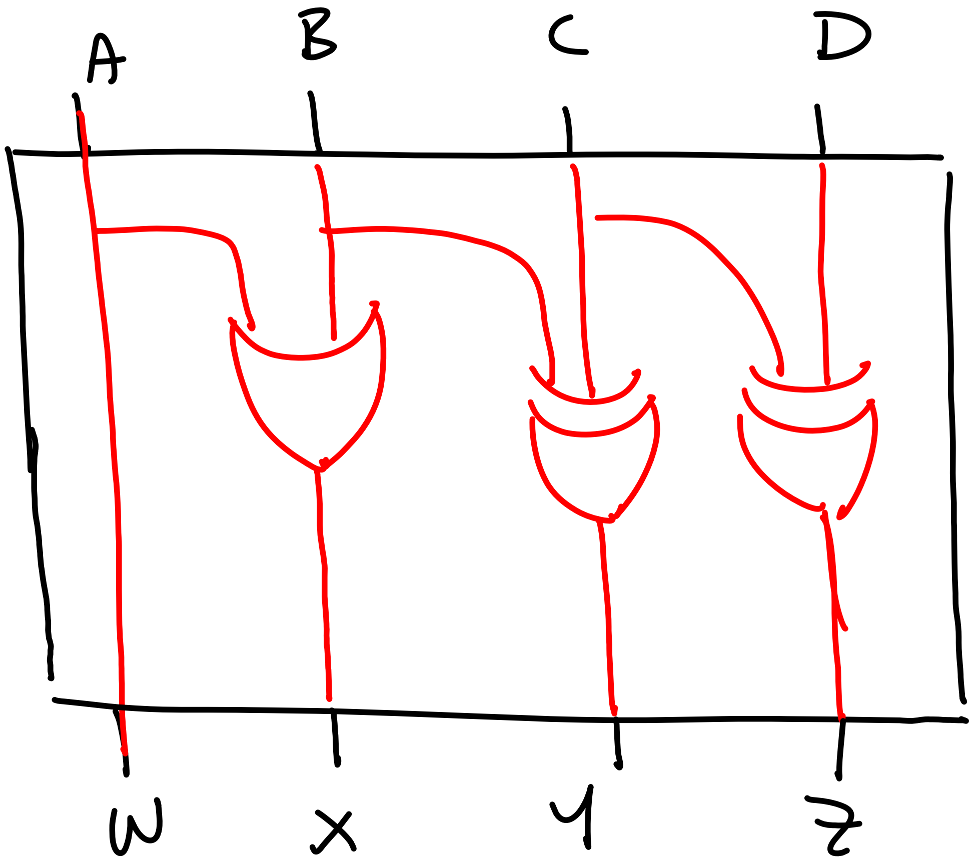
$$= B \oplus C$$

Z

AB \ CD	00	01	11	10
00		1		1
01		1		1
11	X	X	X	X
10		1	X	X

$$Z = \bar{C}D + C\bar{D}$$

$$Z = C \oplus D$$



Note

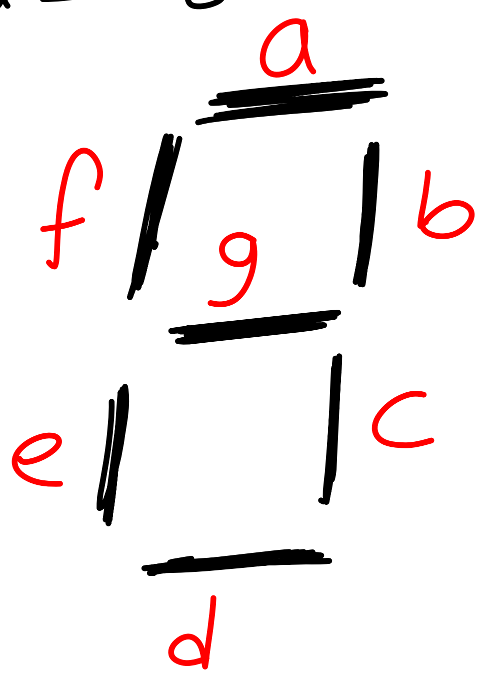
Design procedure

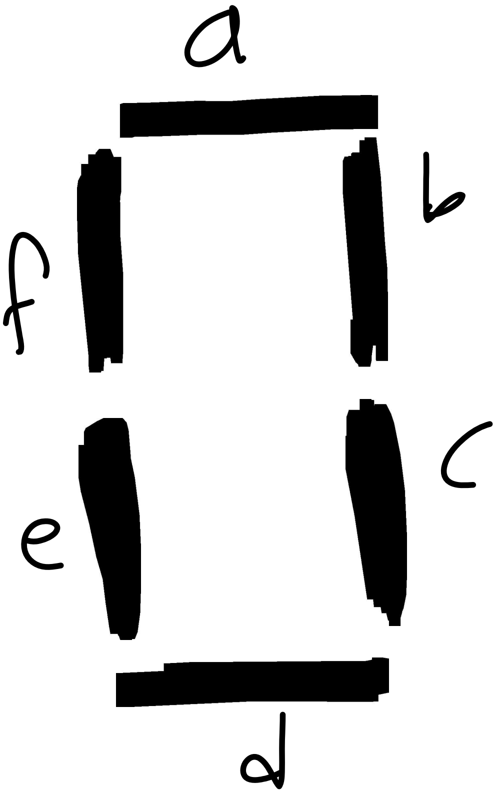
1. problem description
2. truth table
   - Determine No. of inputs
   - Determine No. of output
   - based on problem description determine output
3. Go To Karnaugh
4. implement

Prob

A seven_segment display decoder takes a 4-bit data input, $D_{3:0}$, and produces seven outputs control light emitting diodes to display a digit from 0 to 9. the seven outputs are often called segments a through g or $S_a - S_g$ as defined in fig. below

write a truth table for the outputs and use k-map to boolean equations for outputs $S_a - S_b$

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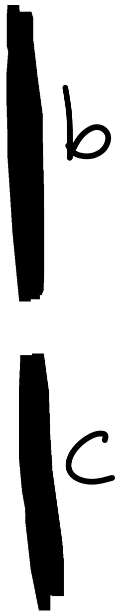
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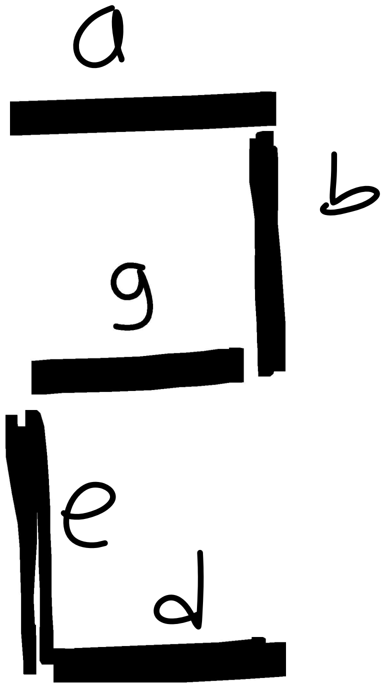
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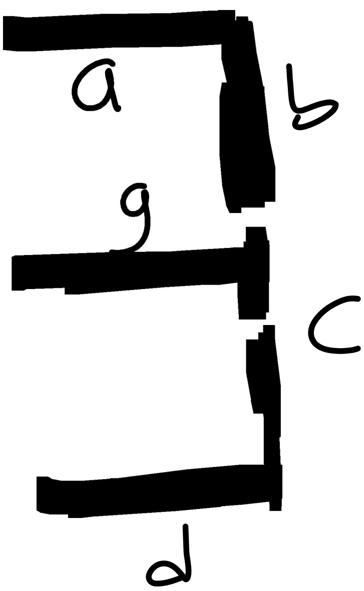
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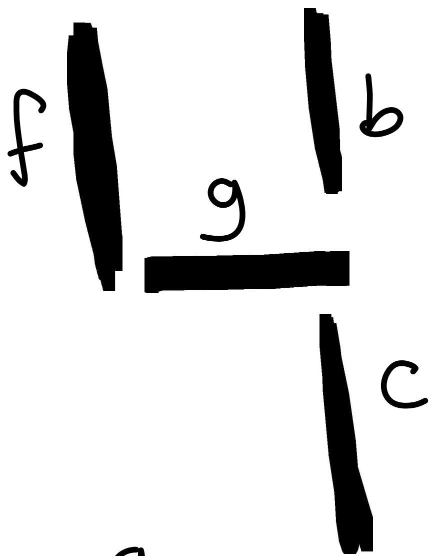
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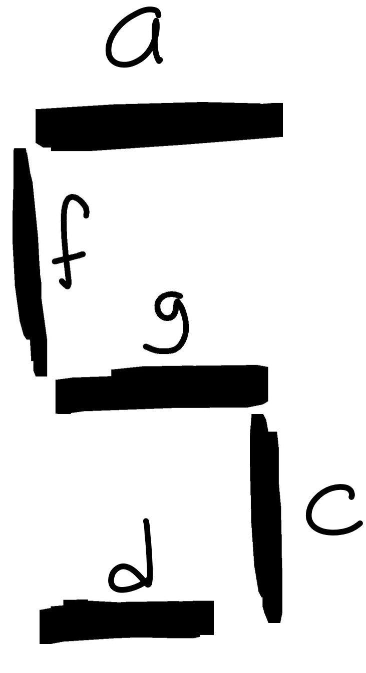
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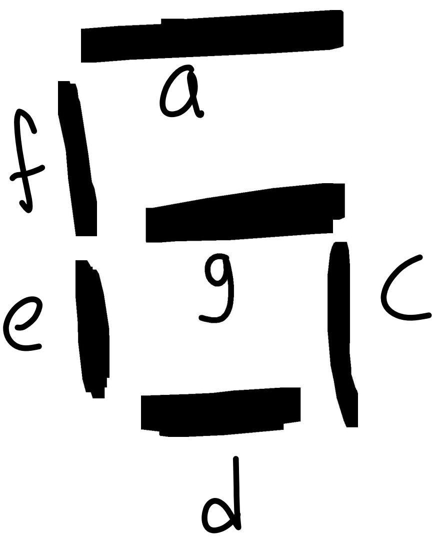
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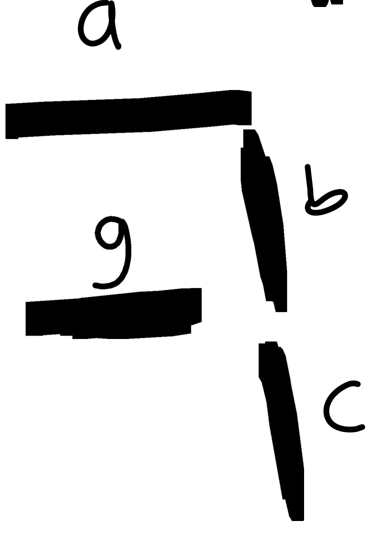
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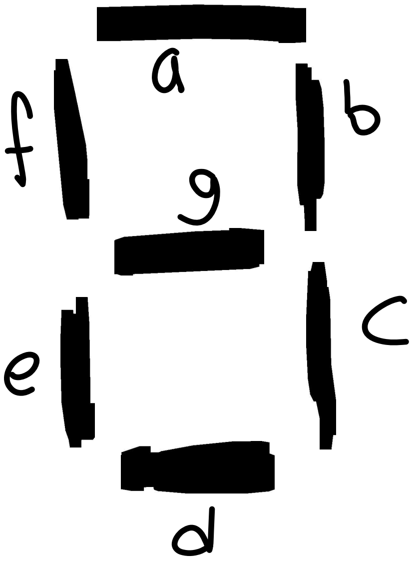
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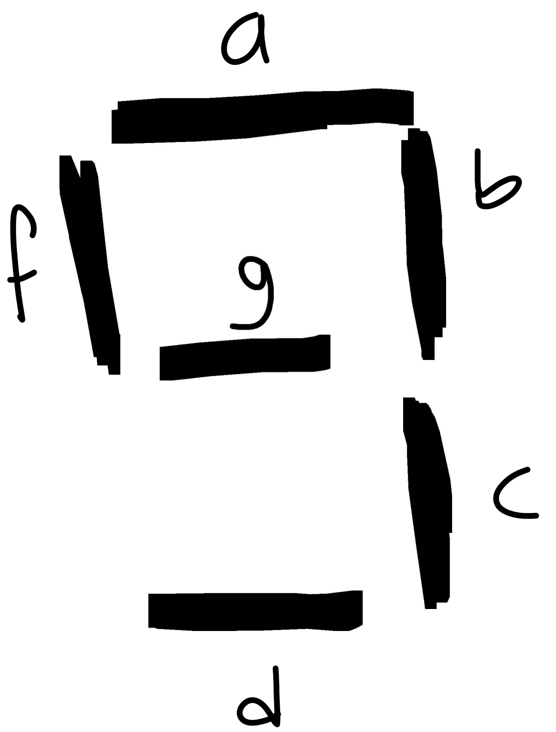
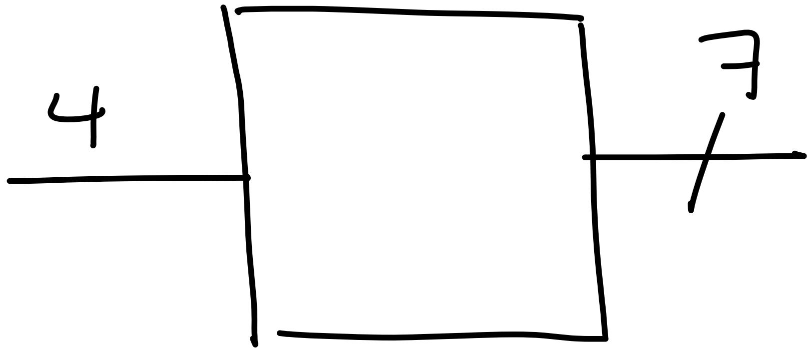


Sol.



D_3	D_2	D_1	D_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1

⇓ continue