Probability

Counting Methods - 01

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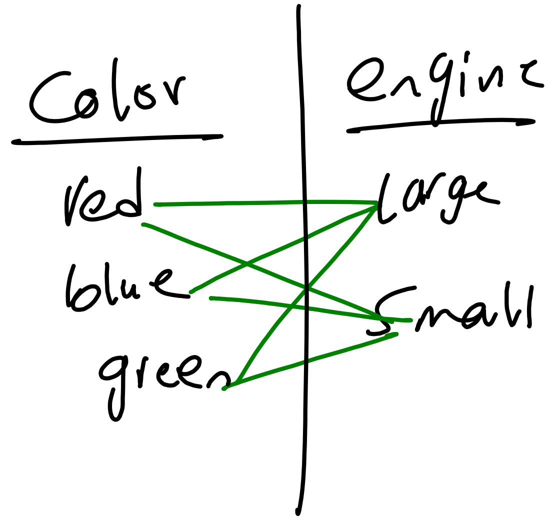
Counting

- When **computing probabilities**, it is sometimes **necessary** to **determine the number of outcomes** in a sample space

Example

- A certain make of automobile is available in any of **three colors**: red, blue, or green, and comes with either a **large or small engine**. In **how many ways** can a buyer choose a car?

	Red	Blue	green
large	Red large	Blue large	
Small			Green Small



outcome = 6

$$\left.\begin{array}{l}\#\ \text{color} = \underline{\underline{n_1}} \\ \#\ \text{engine} = \underline{\underline{n_2}}\end{array}\right\}\ \boxed{n_1 * n_2}$$

Fundamental principle of counting

- Assume that k operations are to be performed.
- If there are n_1 ways to perform the first operation,
- and if for each of these ways there are n_2 ways to perform the second operation,
- and if for each choice of ways to perform the first two operations there are n_3 ways to perform the third operation,
- and so on,
- then the total number of ways to perform the sequence of k operations is $n_1 n_2 \cdots n_k$.

$\boxed{n_1 \times n_2 \times \cdots \times n_k}$

Example 01

- When ordering a certain type of computer, there are **3** choices of hard drive, **4** choices for the amount of memory, **2** choices of video card, and **3** choices of monitor. In how many ways can a computer be ordered?

The total number of ways to order a computer is

$$3 * 4 * 2 * 3 = \boxed{72}$$

Permutations

- A **permutation** “ التبديل ”is an ordering of a collection of objects. For example, there are **six permutations** of the letters A, B, C:

$3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times \underline{3 \times 2 \times 1} = 24 = 4 * 3!$

Order

Permutation

A, B, C

ABC
ACB
BAC
BCA
CAB
CBA

3 2 1

A — B — C
A — C — B
B — A — C
B — C — A
C — A — B
C — B — A

$= 3 * 2 * 1 = 6 = \boxed{3!}$

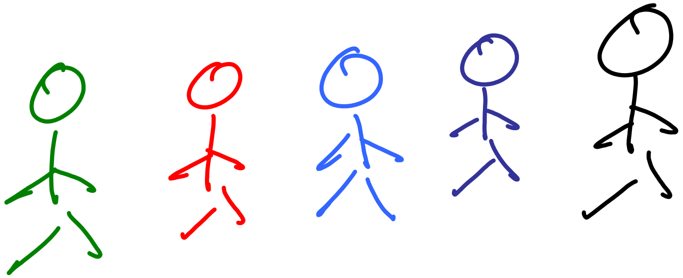
$n! = n * n-1 * n-2 * \dots 1$

Permutations – How to count?

- The **fundamental principle of counting** can be used to determine the number of permutations of any set of objects.
- For example, we can determine the number of permutations of a set of three objects as follows. **There are 3 choices for the object to place first**. After that choice is made, there are **2 choices remaining for the object to place second**. Then there is **1 choice left** for the object to place last.
- Therefore, the total number of ways to order three objects is **(3)(2)(1) = 6**.

Example

- Five people stand in line at a movie theater. **Into how many different orders can they be arranged?**



$$(5) * (4) * (3) * (2) * (1)$$

$$= 5! = \boxed{120}$$

Note…

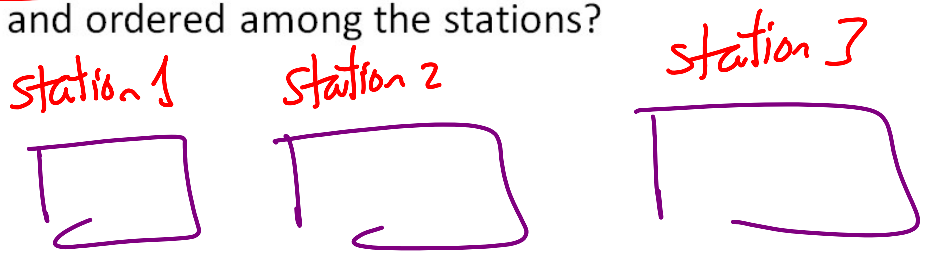
- Sometimes we are interested in counting the **number of permutations of subsets of a certain size** chosen from a larger set

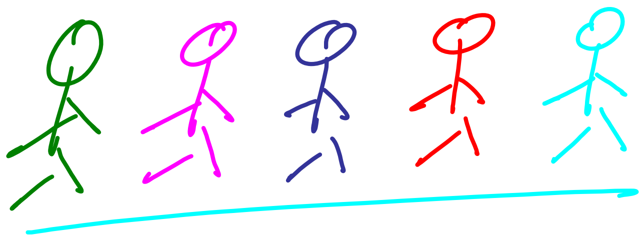
Example

- Five **lifeguards** “ منقذين “ are available for duty one Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations?

lifeguards = 5

Stations = 3





$5 \times 4 \times 3 = \boxed{60}$

Permutations

- The number of permutations of k objects chosen from a group of n objects is

$$\frac{n!}{(n-k)!}$$

$n \rightarrow$ life guards

$k \rightarrow$ No of stations

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy