Probability

Counting Methods – Combinations

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Combinations

- In some cases, when choosing a set of objects from a larger set, we **don't care about the ordering** of the chosen objects; we **care only which objects are chosen**.
- For example, we **may not care which lifeguard occupies which station**; we might **care only which three lifeguards are chosen**.
- **Each distinct group of objects that can be selected, without regard to order, is called a combination** " مجموعات "

3 A B C 2

ABC
ACB
BAC
BCA
CAB
CBA

~~AB~~
AC
BA
BC
~~CA~~
~~CB~~

$$\# \quad \frac{3!}{1!} = \boxed{6}$$

$$\frac{n!}{k!\,(n-k)!} = \frac{3!}{2!\,(1!)} = \boxed{3}$$

Combinations

- In that example, we showed that there are **60 permutations of three objects chosen from five**. Denoting the objects, A, B, C, D, E, the figure presents a list of all 60 permutations.

ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE
ACB	ADB	AEB	ADC	AEC	AED	BDC	BEC	BED	CED
BAC	BAD	BAE	CAD	CAE	DAE	CBD	CBE	DBE	DCE
BCA	BDA	BEA	CDA	CEA	DEA	CDB	CEB	DEB	DEC
CAB	DAB	EAB	DAC	EAC	EAD	DBC	EBC	EBD	ECD
CBA	DBA	EBA	DCA	ECA	EDA	DCB	ECB	EDB	EDC

Combinations

$\boxed{10}$

6 permutations

1 distinct Combination A, B, C

permutations = 6

1 distinct Combination A, B, D

$$= \frac{n!}{k!\,(n-k)!}$$

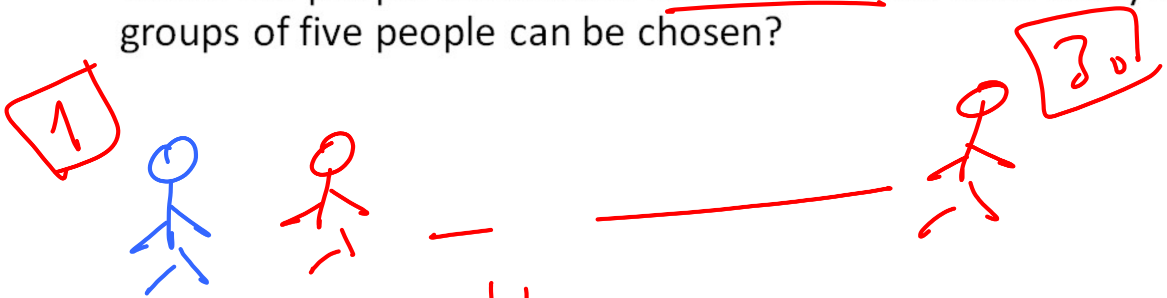
Combinations

- The number of combinations of k objects chosen from a group of n objects is
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Example

- At a certain event, 30 people attend, and 5 will be chosen at random to receive door prizes. The prizes are all the same, so the order in which the people are chosen does not matter. How many different groups of five people can be chosen?



5 persons

$$= \frac{30!}{5!\,(25)!}$$

$$= \frac{30 \times 29 \times 28 \times 27 \times 26}{5 \times 4 \times 3 \times 2} = \underline{\underline{142{,}506}}$$

Example

- Assume that in a class of 12 students, a project is assigned in which the students will work in groups. Three groups are to be formed, consisting of five, four, and three students. We can calculate the number of ways in which the groups can be formed as follows

① $\binom{12}{5} = \frac{12!}{5!\,7!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$

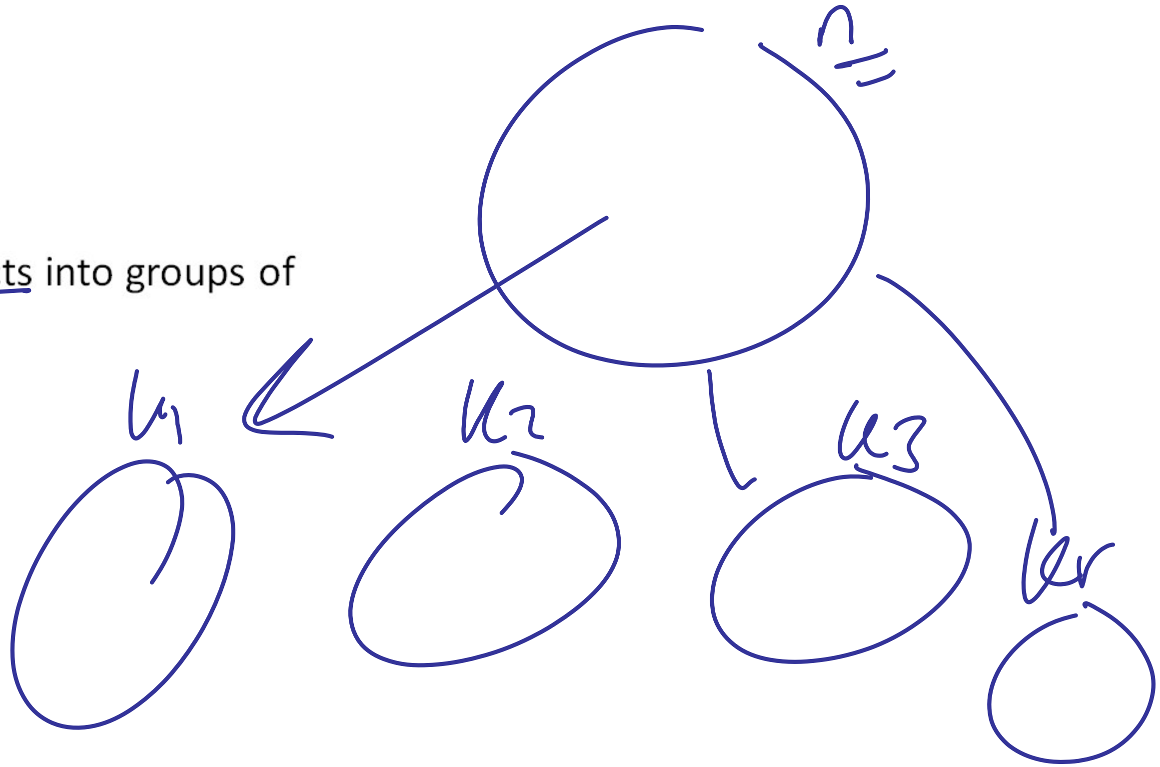
② $\binom{7}{4} = \frac{7!}{4!\,3!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$

$= \boxed{27,720}$

Example

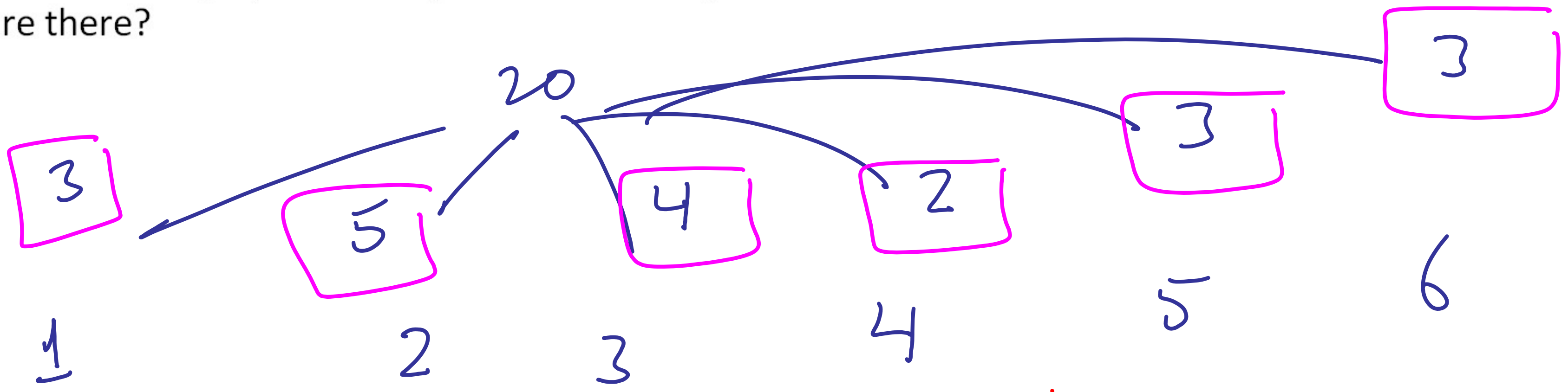
- The number of ways of dividing a group of n objects into groups of $k_1, \ldots, k_r$ objects, where $k_1 + \cdots + k_r = n$, is

$$\frac{n!}{k_1! \ldots k_r!}$$



Example

- A die is rolled 20 times. Given that three of the rolls came up 1, five came up 2, four came up 3, two came up 4, three came up 5, and three came up 6, how many different arrangements of the outcomes are there?



$$\frac{20!}{3!\,5!\,4!\,2!\,3!\,3!} = 1.955 \times 10^{12}$$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Khan Academy