Probability

Conditional Probability

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Conditional Probability

- A **sample space** contains all the possible outcomes of an experiment.
- Sometimes we obtain some **additional information** about an experiment that tells us that the **outcome comes** from a certain part of the sample space.
- **A probability that is based on a part of a sample space** is called a conditional probability

Unconditional Probability

Length	Diameter: Too Thin	Diameter: OK	Diameter: Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- P(diameter OK) ?
- This probability is called the unconditional probability, since it is based on the entire sample space.

1000

$$P(\text{diameter ok}) = \frac{928}{1000}$$

Conditional Probability

	Diameter		
Length	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- Now **assume that a rod is sampled**, and its **length** is measured and found to **meet the specification**. What is the probability that the diameter also meets the specification?
- **knowledge** that the **length meets the specification reduces the sample space** from which the rod is drawn

$$P(\text{diameter ok} \mid \text{length is ok}) = \frac{900}{942}$$

Example

- P(diameter OK | length too long)?

Length	Diameter: Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

$$P(\text{diameter ok} \mid \text{length too long}) = \frac{25}{40} = 0.625$$

$$P(\underbrace{\text{diameter ok}}_{A} \mid \underbrace{\text{length too long}}_{B}) = \frac{25}{40}$$

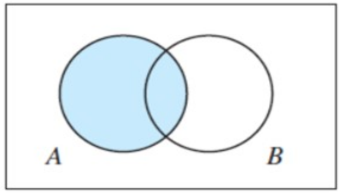
$$= \frac{25/1000}{40/1000}$$

$$= \frac{P(\text{diameter ok and length too long})}{P(\text{length too long})}$$

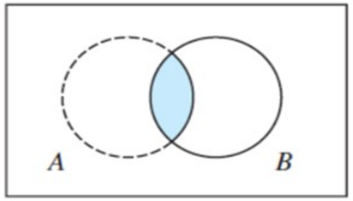
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid \underline{\underline{B}}) = \frac{P(A \cap B)}{P(B)}$$

Venn Diagram



$P(A)$



$P(A \mid B)$

Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the side, given that it has a flaw on top?**

$P(\text{on side}) = 0.02 \Rightarrow A$

$P(\text{on top}) = 0.03 \Rightarrow B$

$P(\text{on top and on side}) = 0.01$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.01}{0.03} = 1/3 = 0.333$$

Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the top, given that it has a flaw on the side?**

$P(\text{on side}) = 0.02 \rightarrow B$

$P(\text{on top}) = 0.03 \rightarrow A$

$P(\text{on side and on top}) = 0.01$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.01}{0.02} = 0.5$$

Independent Events

- Sometimes the **knowledge that one event has occurred does not change the probability that another event occurs**.
- In this case the **conditional and unconditional** probabilities are the **same**, and the **events are said to be independent**

fair coin

H T

$P(H) = 0.5$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy