

Probability

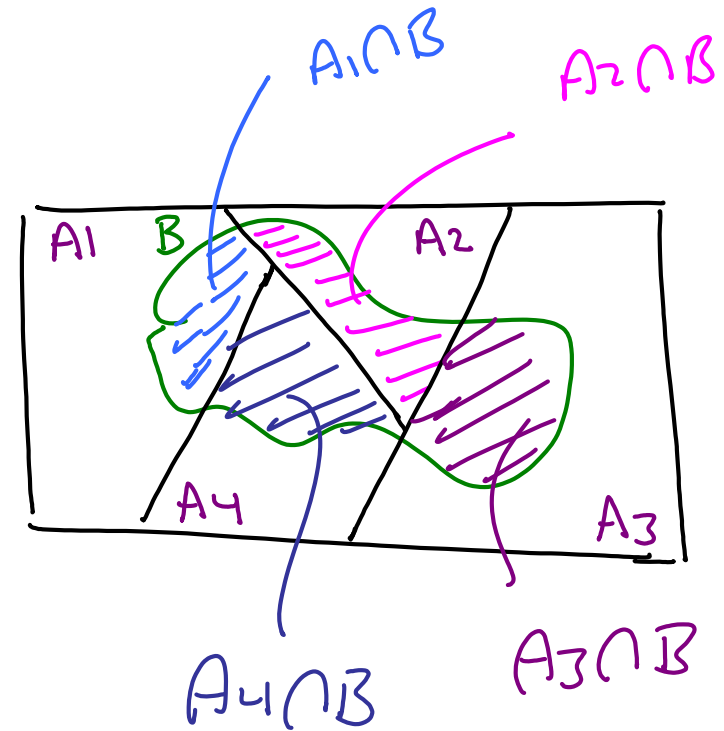
Law of Total Probability| Conditional Probability

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| Intro...

- Assume a sample space contains the events A_1, A_2, A_3 , and A_4 . These events are mutually exclusive, since no two overlap.
- They are also exhaustive, which means that their union covers the whole sample space.
- Each outcome in the sample space belongs to one and only one of the events A_1, A_2, A_3, A_4



$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup (A_3 \cap B) \cup (A_4 \cap B)$$

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) + P(A_4 \cap B)$$



$$\therefore P(A \cap B) = P(B|A) P(A) \Rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B) = P(B|A_1) P(A_1) + P(B|A_2) P(A_2) + P(B|A_3) P(A_3) + P(B|A_4) P(A_4) + \dots$$

| Example

- Customers who **purchase a certain make of car** can order an **engine** in any of **three sizes**. Of all cars sold, 45% have the smallest engine, 35% have the medium-sized one, and 20% have the largest. Of cars with the smallest engine, 10% fail an emissions test within two years of purchase, while 12% of those with the medium size and 15% of those with the largest engine fail.
- What is the probability that a randomly chosen car will fail an emissions test within two years?

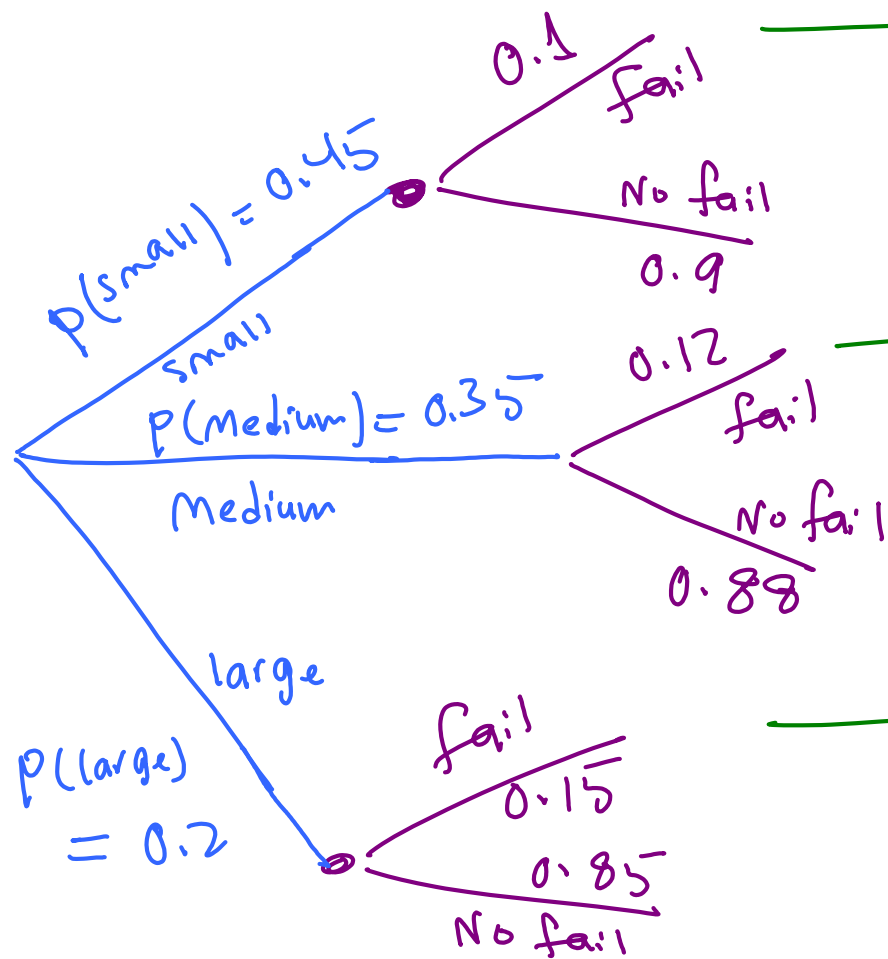
$$\begin{aligned} p(\text{Small}) &= 0.45 & \Rightarrow P(\text{emission fail}) &= \underline{0.1} \\ p(\text{Medium}) &= 0.35 & \Rightarrow P(\text{emission fail}) &= \underline{0.12} \\ p(\text{Large}) &= 0.2 & \Rightarrow P(\text{emission fail}) &= 0.15 \end{aligned}$$

B
↓
P(emission fail)

$$P(B) = P(B | \text{small}) P(\text{small}) + P(B | \text{medium}) \times P(\text{medium}) \\ + P(B | \text{large}) \times P(\text{large})$$

$$= 0.1 \times 0.45 + 0.12 \times 0.35 + 0.15 \times 0.2$$

$$= \underline{\underline{0.117}}$$



$$P(\text{fail} | \text{small}) \times P(\text{small}) \\ = 0.1 \times 0.45 = \boxed{0.045}$$

$$= \boxed{0.042}$$

$$= \boxed{0.030}$$

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy