Probability

Bayes' Rule| Conditional Probability

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Bayes' Rule

- If **A and B are two events**, we have seen that in most cases $P(A|B) \neq P(B|A)$.

$$P(A|B) = \frac{\boxed{P(A \cap B)}}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = \underline{P(B|A)\, P(A)}$$

$$\therefore P(A|B) = \frac{P(B|A)\, P(A)}{\underline{P(B)}}$$

$$P(B) = P(B|A)\, P(A) + P(B|A^c)\, P(A^c)$$

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B|A)\, P(A) + P(B|A^c)\, P(A^c)}$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A^c)\,P(A^c)}$$

Example - 01

- The **proportion** of people in a given community who have a certain **disease** is **0.005**. A test is available to diagnose the disease. If a person **has the disease**, the probability that the test will produce a **positive** signal is **0.99**. If a person **does not have the disease**, the probability that the **test will produce a positive signal is 0.01**.

If a person tests positive, what is the probability that the person actually has the disease?

$$\left\{ \begin{array}{l} P(D) = 0.005 \\ P(+|D) = 0.99 \\ P(+|ND) = 0.01 \end{array} \right.$$

$P(D|+) = ?$

$$P(D|+) = \frac{P(+|D) * P(D)}{P(+|D) * P(D) + P(+|ND) * P(ND)} = \frac{0.99 * 0.005}{0.99 * 0.005 + 0.01 * 0.995}$$

$$= 0.332$$

Example - 02

$A_1 \to$ small
$A_2 \to$ medium
$A_3 \to$ large
$B \to$ fail

- Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, **45%** have the **smallest** engine, **35%** have the **medium-sized** one, and **20%** have the **largest**. Of cars with the **smallest** engine, **10% fail** an emissions test within two years of purchase, while **12%** of those with the **medium** size and **15%** of those with the **largest** engine fail.

A **record for a failed emissions** test is chosen at random. **What is the probability that it is for a car with a small engine**?

$P(A1) = 0.45 \to P(B|A_1) = 0.1$

$P(A2) = 0.35 \to P(B|A_2) = 0.12$

$P(A3) = 0.2 \to P(B|A3) = 0.15$

$$P(\underbrace{A_1}_{\text{small}} | \underbrace{B}_{\text{fail}}) = \frac{P(B|A_1)P(A_1)}{P(B)}$$

$$= \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A1) * P(B|A_2)P(A_2) + P(B|A3)P(A3)}$$

$$P(A_1|B) = \frac{\overset{0.1}{P(\cancel{B|}A_1)}\ \overset{0.45}{P(A_1)}}{\underset{0.1}{P(B|A_1)}\ \underset{0.45}{P(A_1)} + \underset{0.12}{P(B|A_2)}\ \underset{0.35}{P(A_2)} + \underset{0.15}{P(B|A_3)}\ \underset{0.2}{P(A_3)}}$$

Small

fail

$$= \frac{0.1 * 0.45}{0.1 * 0.45 + 0.12 * 0.35 + 0.15 * 0.2}$$

$$= \underline{\underline{0.385}}$$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy