Probability

Reliability Analysis| Conditional Probability

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Reliability analysis

- **Reliability analysis** is the branch of engineering concerned with **estimating the failure rates of systems**.

Example - 01

$P(A) = 0.98$

$P(B) = 0.95$

توالی

- A system contains **two components, A and B, connected in series** as shown in the following diagram. The system will **function only if both** components function. The probability that **A functions** is given by $\boldsymbol{P(A) = 0.98}$, and the probability that **B functions** is given by $\boldsymbol{P(B) = 0.95}$. Assume that **A and B function independently**.

Find the probability that the system functions.

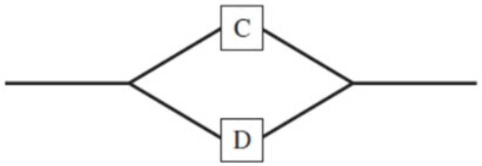
$\cap$

$P(\text{System functions}) = P(A \text{ function and } B \text{ function})$

$= P(A) * P(B) = 0.98 * 0.95$

$= 0.931$

Example - 02



- A system contains **two components, C and D**, connected in **parallel** as shown in the following diagram.
- The system will **function if either C or D functions**. The probability that **C functions is 0.90**, and the probability that **D functions is 0.85**. **Assume C and D function independently**.

Find the probability that the system functions.

$P(C) = 0.9$

$P(D) = 0.85$

$P(\text{system functions})$

$= P(C \cup D)$

$= P(C) + P(D) - P(C \cap D)$

$= P(C) + P(D) - P(C) * P(D)$

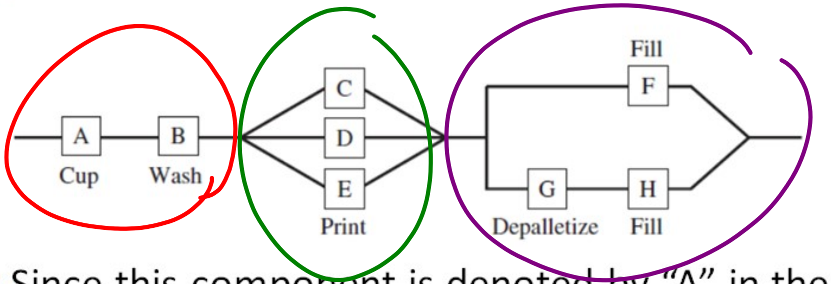
$= 0.9 + 0.85 - 0.9 * 0.85$

$= 0.985$

Note

- The reliability of more complex systems can often be determined by **decomposing** the system into a **series of subsystems**, each of which contains components connected either **in series or in parallel**

Example 03



- Since this component is denoted by "A" in the diagram, we will express this probability as $\boldsymbol{P(A) = 0.995}$. Assume that the other process components have the following probabilities of functioning successfully during a one-day period: $P(B) = 0.99$, $P(C) = P(D) = P(E) = 0.95$, $P(F) = 0.90$, $P(G) = 0.90$, $P(H) = 0.98$. Assume the components **function independently**.

Find the probability that the process functions successfully for one day

$P(\text{Subsystem } 1 \text{ function}) = P(A \cap B) = P(A) * P(B)$
$= 0.995 * 0.99 = 0.985$

$P(\text{subsystem } 2 \text{ function})$
$= 1 - P(\text{subsystem } 2 \text{ fail})$
$= 1 - P(C^c \cap D^c \cap E^c)$
$= 1 - P(C^c) * P(D^c) * P(E^c)$
$= 1 - (0.05)^3$
$= 0.999875$

$P(\text{subsystem } 3 \text{ funtions})$
$= P(F \cup (G \cap H))$

$$P(\text{Subsystem } 3 \text{ functions}) = P(F \cup \underline{(G \cap H)})$$

$$= P(F) + P(G \cap H) - P(F \cap G \cap H)$$

$$= P(F) + P(G) * P(H) - P(F) * P(G) * P(H)$$

$$= 0.9 + 0.9 * 0.98 - 0.9 * 0.9 * 0.98$$

$$= 0.988$$

P(Subsystem 1, 2, 3 all functions)

$$P(\text{system functions}) = 0.973$$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Khan Academy