Probability

Random Variable - Examples

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

Example - 01

- When a student attempts to **log on to a computer time-sharing** system, **either all ports are busy (F)**, in which case the student will **fail to obtain access**, or else there is **at least one port free (S)**, in which case the student will be successful in accessing the system. With $\mathcal{S}\{\boldsymbol{S}, \boldsymbol{F}\}$, define random variable X by $X(S) = 1, (F) = 0$

Success

fail

$X(s) = 1$

$X(f) = 0$

Example - 02

- Consider the experiment in which a **telephone number in a certain area code is dialed using a random number dialer** (such devices are used extensively by **polling** organizations), and define random variable Y by

$$Y = \begin{cases} 1 & \text{if the selected number is unlisted} \\ 0 & \text{if the selected number is listed in the directory} \end{cases}$$

5282966

exist — listed

$Y(5282966) = 0$

7727350

unlisted

$Y(7727350) = 1$

Random Variable

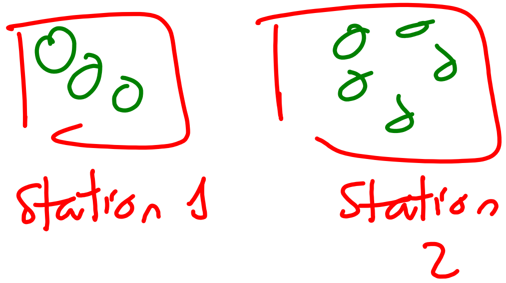
0, 1

Possible values

Bernoulli random variable

Example - 03

- In an experiment in which the **number of pumps in use at each of two gas stations was determined**. Define random variable X, Y, and U by
  - X the total number of pumps in use at the two stations
  - Y the difference between the number of pumps in use at station 1 and the number in use at station 2
  - U the maximum of the numbers of pumps in use at the two stations



S (2, 3)

Pump station 1

Pump at station 2

$X(2,3) = 5$

$Y(2,3) = -1$

$U(2,3) = 3$

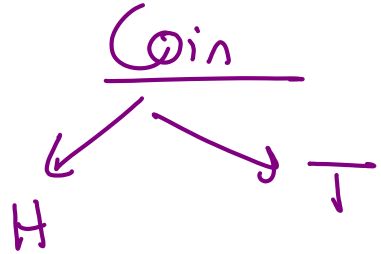
Example - 04

- Each of the random variables of the above examples can assume only a **finite number of possible values**. This need not be the case
- the experiment in which batteries were examined until a good one (S) was obtained. The sample space was $\mathcal{S}\ \{S, FS, FFS, \ldots\}$. Define random variable X by

X: # batteries examined befor the experiment terminates

$$\begin{cases} X(S) = 1 \\ X(FS) = 2 \\ X(FFFS) = 4 \\ \cancel{X(FSFS) =} \end{cases}$$

Example - 05



- The experiment of tossing a coin twice. Let X represent number of heads appearing
  - X is a random variable with values 0, 1, 2
  - $X(T,T) = 0$
  - $X(H,T) = 1$

$X(TT) = 0$

$X(TH) = 1$

$X(HT) = 1$

$X(HH) = 2$

OutCome	X
T T	0
T H	1
H T	1
H H	2

x	$P(X=x)$
0	1/4
1	2/4
2	1/4

Example - 06

- **Computer chips** often contain surface **imperfections**. For a certain type of computer chip, **9% contain no imperfections**, **22%** contain **1** imperfection, **26%** contain **2** imperfections, **20%** contain **3** imperfections, **12%** contain **4** imperfections, and the remaining **11%** contain **5** imperfections. Let Y represent the number of imperfections in a randomly chosen chip.
- What are the possible values for Y? Is Y discrete or continuous? Find $P(Y = y)$ for each possible value y.

$P(Y=3) = 0.2$

$P(Y=5) = 0.11$

$Y \Rightarrow$ # imperfections

possible values

0, 1, 2, 3, 4 and 5

discrete Random Variable

Example - 07

- A certain type of **magnetic disk** must function in an environment where it is **exposed to corrosive gases**. It is known that **10%** of all such disks have **lifetimes less than or equal to 100 hours**, **50%** have lifetimes **greater than 100 hours but less than or equal to 500 hours**, and **40%** have lifetimes **greater than 500 hours**.
- Let Z represent the number of hours in the lifetime of a randomly chosen disk. Is Z **continuous or discrete**? Find $P(Z \le 500)$. Can we compute all the probabilities for Z? Explain.

$P(Z \le 100) = 0.1$

$P(100 < Z \le 500) = 0.5$

$P(Z > 500) = 0.4$

Random variable ⇒ Not discrete

Continuous

$P(Z \le 500) = 0.6$

Discrete – Continuous Random Variable

- When the probabilities pertaining to a random variable are known, we usually do not think about the sample space; we just focus on the probabilities
- There are two important types of random variables, **discrete and continuous.**
  - A **discrete random variable** is one whose possible values form a **discrete set**.
  - The possible values of a **continuous random variable** always contain an **interval**, that is, all the points between some two numbers.

Discrete – Continuous Random Variable

- Discrete
  - Finite set
  - Infinite "But counted"
- continuous
  - Interval

Probability Mass Function

- **The number of flaws in a 1-inch length of copper wire** manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**. Let X be the number of flaws in a randomly selected piece of wire. Then
  - $P(X = 0) = 0.48 \quad P(X = 1) = 0.39$
  - $P(X = 2) = 0.12 \quad P(X = 3) = 0.01$
- The list of possible values **0, 1, 2, 3**, **along** with the **probabilities** for each, provide a **complete description of the population** from which X is drawn. This description has a name—the **probability mass function**

No flaws ⇒ 48 %

one flaw ⇒ 39 %

two flaws ⇒ 12 %

three flaws ⇒ 1 %

$X \rightsquigarrow 0, 1, 2, 3$

Probability distribution

$P(X=0) = 0.48$ $\quad P(X=2) = 0.12$

$P(X=1) = 0.39$ $\quad P(X=3) = 0.01$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy