Probability

Probability Mass Function | Random Variable

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Intro

- It is common for the possible values of a discrete random variable to be a set of integers.
- For any discrete random variable, if we specify the **list of its possible values along with the probability** that the random variable takes on each of these values, then we have **completely described** the **population** from which the random variable is sampled

Example - 01

- Suppose, for example, that a business has just purchased **four laser printers**, and let X be the number among these that **require service during the warranty period**.

$X \Rightarrow 0, 1, 2, 3, 4 \Rightarrow$ discrete values $\Rightarrow$ discrete Random Variable

$0, 1, 2, 3, 4$

$P(X=0) = P(0) = ?$

$P(X=1) = P(1) =$

$p(x) = P(X = x)$

Example - 02

- **The number of flaws in a 1-inch length of copper wire** manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**.

$X \Rightarrow \#$ flaws

$P(X=0) = P(0) = 0.48$

$P(X=2) = P(2) = 0.12$

$P(X=1) = P(1) = 0.39$

$P(3) = 0.01$

Sum = 1

probability mass function

PMF

Probability distribution

Probability Mass Function

PMF

- The **probability mass function** of a discrete random variable X is the function $\boldsymbol{p}(\boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{X} = \boldsymbol{x})$.
- The probability mass function is sometimes called the **probability distribution**.

Probability Mass Function

- Note that if the **values of the probability mass function are added over all the possible values of X, the sum is equal to 1**.

Example - 03

- A certain gas station has **six pumps**. Let X denote the **number of pumps that are in use** at a particular time of day. Suppose that the **probability distribution of X** is as given in the following table

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

$\Rightarrow 1 \Rightarrow$ PMF

$X \rightarrow$ # Pumps

Example - 03

- The probability that at most 2 pumps are in use?
- $p(x \geq 3)$?
- The probability that between 2 and 5 pumps inclusive are in use ?

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

$P(x \leq 2) = P(0) + P(1) + P(2) = 0.3$

$P(X \geq 3) = P(3) + P(4) + P(5) + P(6) = 0.7$

$P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.3 = 0.7$

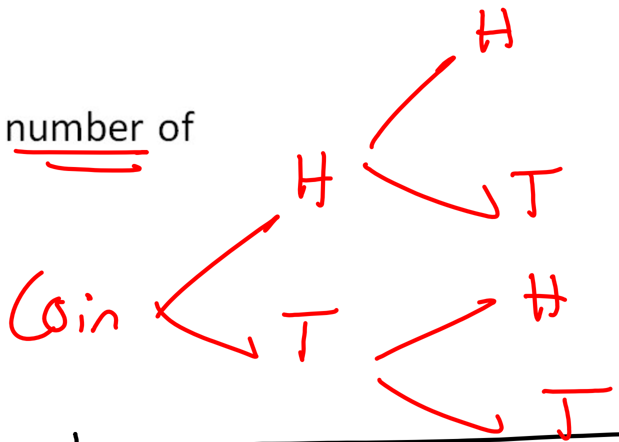
2, 3, 4, 5

$\rightarrow$ 0.75

Example - 04

- The experiment of **tossing a coin twice**. Let X represent number of heads appearing

$X \Rightarrow$ # head

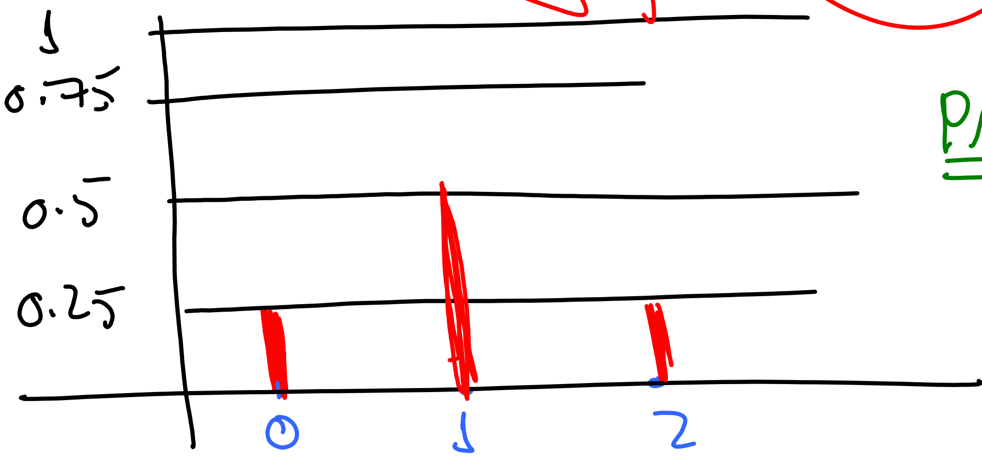


HH
HT
TH
TT

$P(X=0) = P(TT) = 1/4$

$P(X=1) = P(HT, TH) = 2/4$

$P(X=2) = P(HH) = 1/4$



pmf

Example - 05

- Considering the experiment of tossing two dice. The random variable X represents the sum of the two numbers. Find the probability distribution $p(X)$

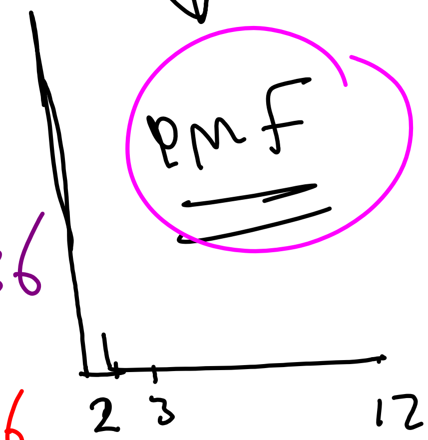
$p(X=2) = 1/36$

$p(X=3) = 2/36$

$p(X=4) = 3/36$

$\vdots$

$p(X=12) = 1/36$



PMF

36

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability Mass Function - Graph

- The **probability mass function can be represented by a graph** in which a **vertical line** is drawn **at each of the possible values** of the random variable. The **heights** of the lines are **equal to the probabilities of the corresponding values**.

Mass = height

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy