Probability

Random Variable [Cumulative Distribution Function]

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

Cumulative Distribution Function

- The **probability mass function** specifies the probability that a random variable is equal to a given value.
- The **cumulative distribution function** specifies the probability that a random variable is less than or equal to a given value

$$F(X=x) = P(X \leq x)$$

$$F(2) = P(X \leq 2)$$

PMF ⇒ Probability Distribution

$P(X=x)$

- x → possible value
- P → defined for all possible values

Example - 01

defect

- The number of flaws in a 1-inch length of copper wire manufactured by a certain process varies from wire to wire. Overall, 48% of the wires produced have no flaws, 39% have one flaw, 12% have two flaws, and 1% have three flaws. Let X be the number of flaws in a randomly selected piece of wire. Let **F(x)** denote the **cumulative distribution function** of the random variable X that represents the number of flaws in a randomly chosen wire. Find F(2). Find F(1.5).

$F(1.5) = P(X \leq 1.5) = P(1) + P(0) = 0.87$

x	0	1	2	3
$P(X=x)$	0.48	0.39	0.12	0.01
$F(X=x)$	0.48	0.87	0.99	1

$F(2) = P(X \leq 2) = P(1) + P(0) + P(2)$

$F(X=x) = P(X \leq x)$

$F(1) = P(X \leq 1)$

$= P(0) + P(1)$

$= 0.48 + 0.39$

$= 0.87$

$F(0) = P(X \leq 0) = P(0)$

Note...

$$F(X=x) = P(X \leq x)$$

- In general, for any discrete random variable X, the **cumulative distribution function F(x) can be computed by summing the probabilities of all the possible values of X that are less than or equal to x**.
- Note that **F(x) is defined for any number x**, not just for the possible values of X

Cumulative Distribution

Not like PMF → defined for all possible values

$P(X=x) = 0$ → Not in possible Values

Example - 02

- Plot the cumulative distribution function F(x) of the random variable X that represents the number of flaws in a randomly chosen wire.

x	0	1	2	3
F(X=x)	0.48	0.87	0.99	1

$$F(0 \le x < 1) = F(0) = P(x \le 0)$$

$$F(1 \le x < 2) = F(1)$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.48 & 0 \le x < 1 \\ 0.87 & 1 \le x < 2 \\ 0.99 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

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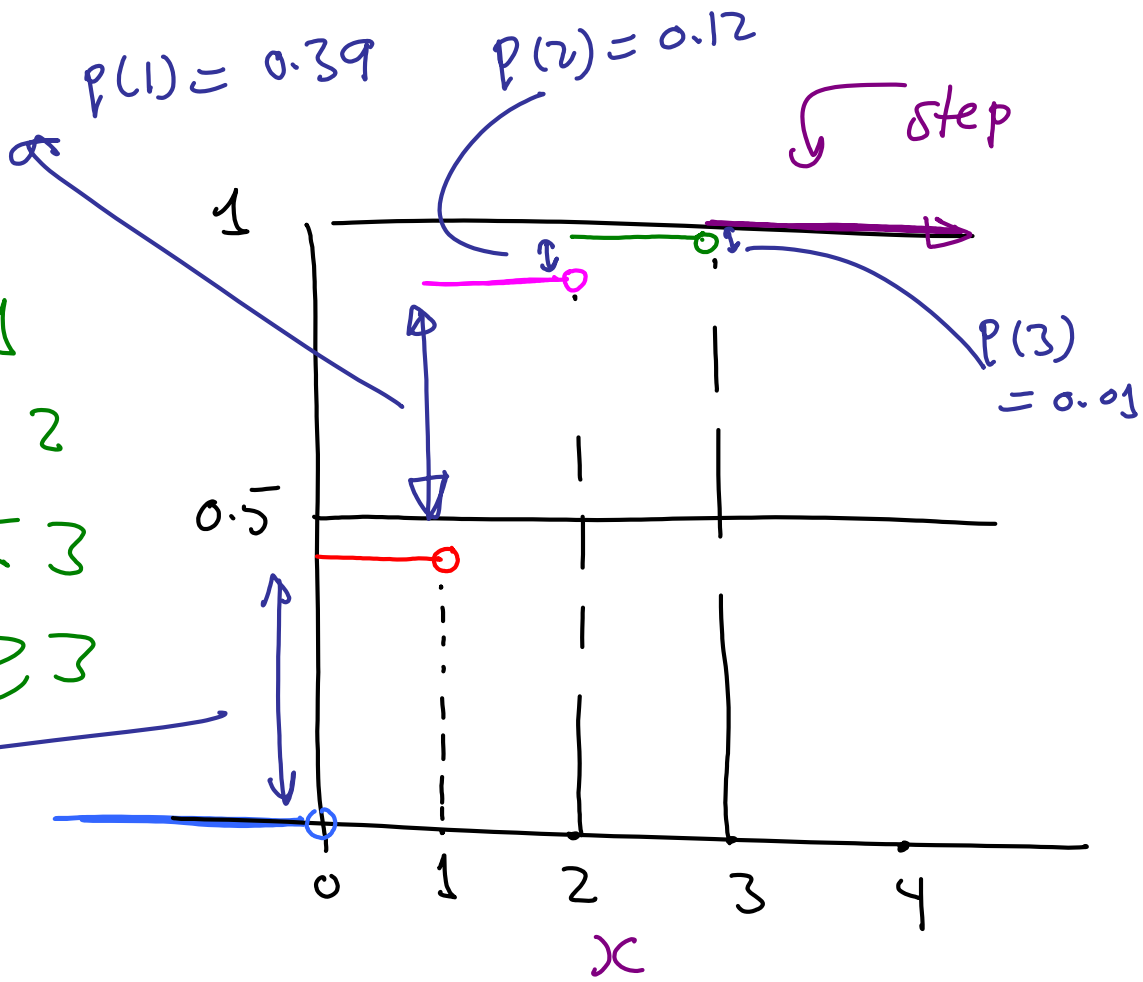

References

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