Probability

Discrete Random Variable [Mean and Variance Examples]

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Population Mean

- Let X be a discrete random variable with probability mass function $p(x) = P(X = x)$.

The mean of X is given by $\boldsymbol{\mu_X = \sum_x xP(X = x)}$

where the sum is over all possible values of X.

Population Variance

$$\sigma_X^2 = \sum_x (x - \mu_X)^2 P(X = x)$$

$$\sigma_X^2 = \sum_x x^2 P(X = x) - \mu_X^2$$

$$\sigma_x^2 = \sum_x x^2 p(X = x) - \mu_x^2$$

Example - 01

- Consider **selecting at random** a student who is among the **15,000** registered for the current term at certain University. Let X = the number of courses for which the selected student is registered, and suppose that X has the following pmf:

x	1	2	3	4	5	6	7
$p(x)$	.01	.03	.13	.25	.39	.17	.02

150 450

- Find the expected value

Population mean

mean

expected value

Expectance

1, 1, 1, . ——, 1, 2, 2, 2 ——

Population

$$\mu_x = \frac{150(1) + 450(2) + \cdots}{15{,}000}$$

$$= \frac{150}{15{,}000} * \boxed{1} + \cdots + \cdots$$

$$= 0.01 * x$$

$$= P(X = x) * x$$

$$\mu_x = 1 * 0.01 + 2 * 0.03 + 3 * 0.13 + 4 * 0.25 + 5 * 0.39 + 6 * 0.17 + 7 * 0.02$$

$$\mu_x = 4.57$$

Example - 02

- Just after birth, **each newborn** child is rated on a **scale** called the **Apgar** scale. The possible ratings are **0, 1, . . . , 10**, with the child's rating determined by **color**, **muscle** tone, **respiratory** effort, **heartbeat**, and **reflex** irritability (the **best possible score is 10**).
- Let X be the Apgar score of a randomly selected child born at a certain hospital **during the next year**, and suppose that the pmf of X is

x	0	1	2	3	4	5	6	7	8	9	10
$p(x)$	.002	.001	.002	.005	.02	.04	.18	.37	.25	.12	.01

- Find the mean value ?

$$\mu_x = 0*0.002 + 1*0.001 + 2*0.002 + 3*0.005 + 4*0.02 + 5*0.04 + 6*0.18 + 7*0.37 + 8*0.25 + 9*0.12 + 10*0.01 = \boxed{7.15}$$

Example - 03

- A resistor in a certain circuit is specified to have a resistance in the range **99 Ω − 101 Ω**. An engineer obtains **two resistors**. The probability that **both of them meet the specification is 0.36**, the probability that exactly **one of them** meets the specification is **0.48**, and the probability that **neither of them** meets the specification is **0.16**. **Let X represent the number of resistors that meet the specification**.
- Find the probability mass function, and the mean, variance, and standard deviation of X.

(1)

x	0	1	2	
$P(X=x)$	0.16	0.48	0.36	$=1$

$$\mu_X = E(x) = 0 * 0.16 + 1 * 0.48 + 2 * 0.36 = 1.2$$

$$\sigma_X^2 = (0-1.2)^2(0.16) + (1-1.2)^2 0.48 + (2-1.2)^2(0.36) = 0.48$$

$$\sigma_X^2 = 0^2(0.16) + (1)^2(0.48) + (2)^2(0.36) - (1.2)^2$$

$$\sigma = \sqrt{\sigma_X^2} = 0.693$$

Prob - 01

- Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

- Find $P(X \leq 2)$.
- Find $P(X > 1)$.
- Find μ_X
- Find σ_X^2

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy