

Probability

Continuous Random Variable

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$X \Rightarrow \# \text{ games}$

Possible Values	probability $P(X=x)$
4	$\longleftrightarrow$
5	$\longleftrightarrow$
6	$\longleftrightarrow$
7	$\longleftrightarrow$

1

probability
distribution



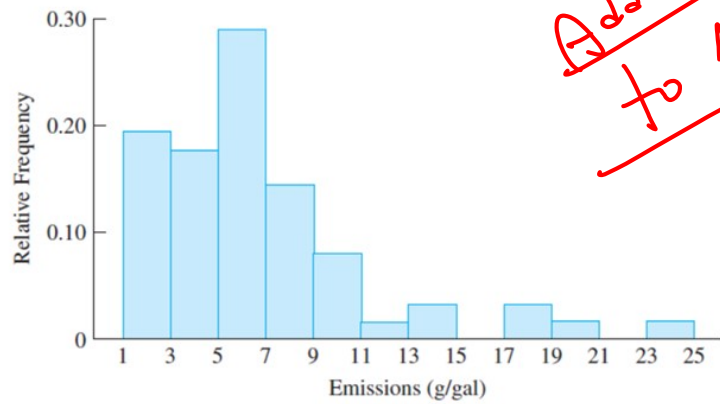
Discrete
Random
Variable

Probability mass function

Histogram

- A **histogram** is a graphic that gives an idea of the shape of a sample
- Frequency presents the numbers of data points that fall into each of the class intervals
- Relative Frequency presents the frequencies divided by the total number of data points
- Density presents the relative frequency divided by the class width

| Histogram



- In some cases, histograms are drawn with **class intervals** of differing widths.

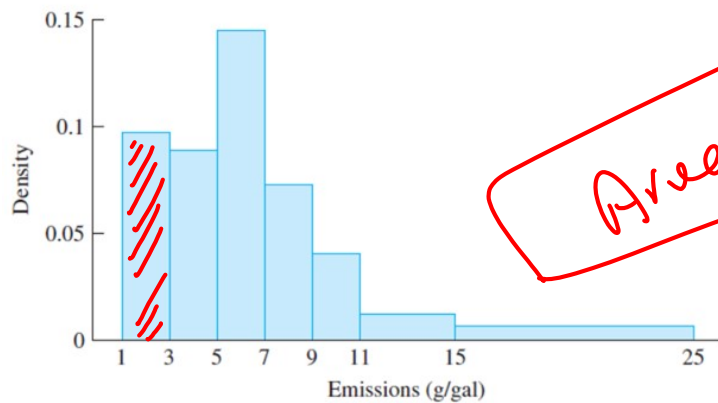
| Histogram – unequal class width

Class Interval (g/gal)	Frequency	Relative Frequency	Density
1-< 3	12	0.1935	0.0968
3-< 5	11	0.1774	0.0887
5-< 7	18	0.2903	0.1452
7-< 9	9	0.1452	0.0726
9-< 11	5	0.0806	0.0403
11-< 15	3	0.0484	0.0121
15-< 25	4	0.0645	0.0065

$$\text{Density} = \frac{\text{Relative freq}}{\text{Class width}}$$

- Because the **class widths vary in size**, the **densities** are **no** longer **proportional to the relative frequencies**. Instead, the densities adjust the relative frequency for the width of the class.

Histogram – unequal class width

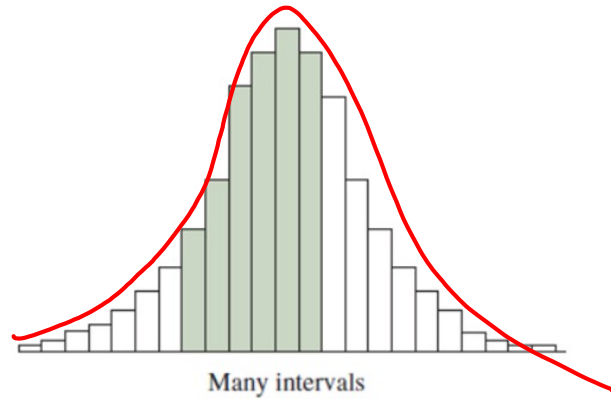


- when the **classes have unequal widths**, the **heights** of the rectangles must be **set equal to the densities**.
- The **areas** of the rectangles then **represent the relative frequencies**

$$\text{Density} = \frac{\text{Relative freq}}{\text{class width}}$$

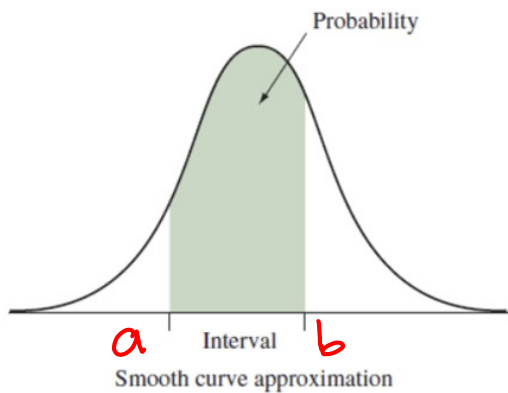
| Probability Density Function (PDF)

- As the **number of intervals increases**, with their **width narrowing**, the **shape** of the histogram **gradually approaches a smooth curve**.
- We'll use such curves to portray probability distributions of continuous random variables



Probability Density Function (PDF)

Continuous
Random Variable



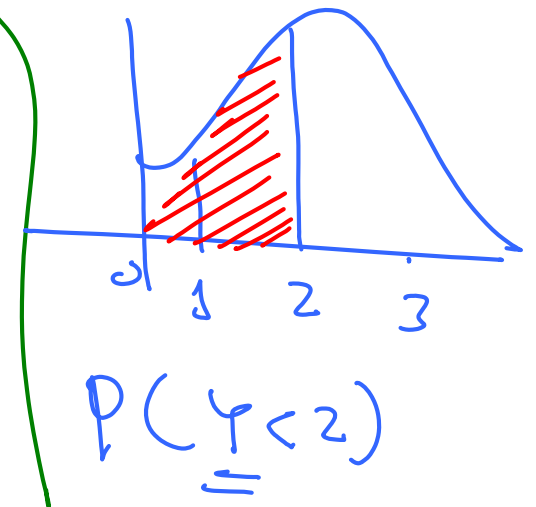
- The **probability** that X falls between any two values a and b is equal to the **area** under the histogram between a and b
- Because the histogram in this case is represented by a **curve**, the **probability** would be found by computing an **integral**

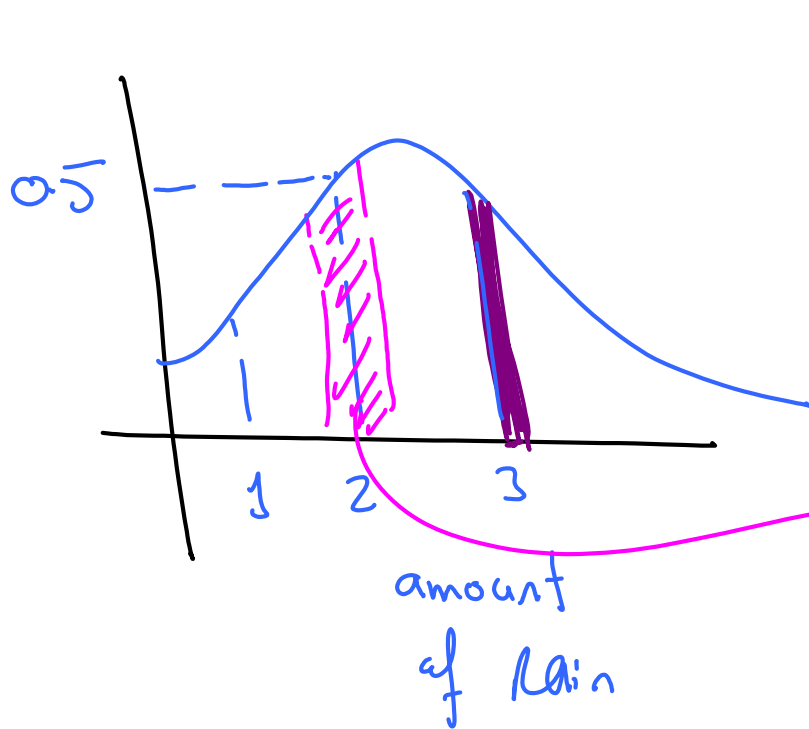
$$\int_a^b f(x) dx = \text{Area under Curve}$$

(a, b)

$$\int_{1.9}^{2.1} f(x) dx$$

$Y =$ Exact amount
of Rain tomorrow





$$P(|Y-2| < \underline{\underline{0.1}})$$

$$\Downarrow$$

$$P(1.9 < Y < 2.1)$$

$$\int_{1.9}^{2.1} f(x) dx$$

$$P(Y=3) = 0$$

$\begin{matrix} b \\ \downarrow \\ a \end{matrix}$

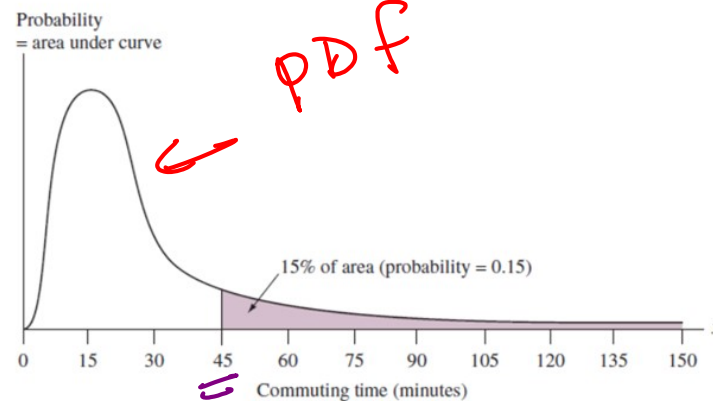
$$P(Y=2) = \underline{\underline{0.5}} \quad \boxed{\times}$$

0 ✓

$$\begin{matrix} 2.0060867 \\ 1.9995969 \end{matrix}$$

Example

- Probability distribution of X = commuting time to work for workers in the United States, based on information from the 2009 American Community Survey



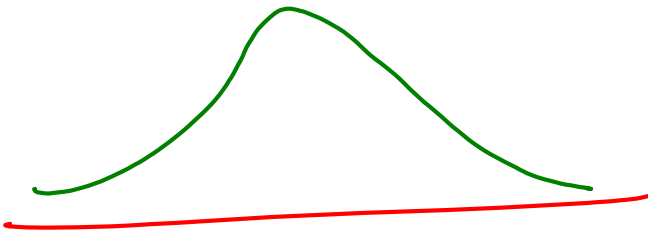
▲ Figure 6.2 Probability Distribution of Commuting Time. The area under the curve for values higher than 45 is 0.15. Question Identify the area under the curve representing the probability of a commute less than 15 minutes, which equals 0.27.

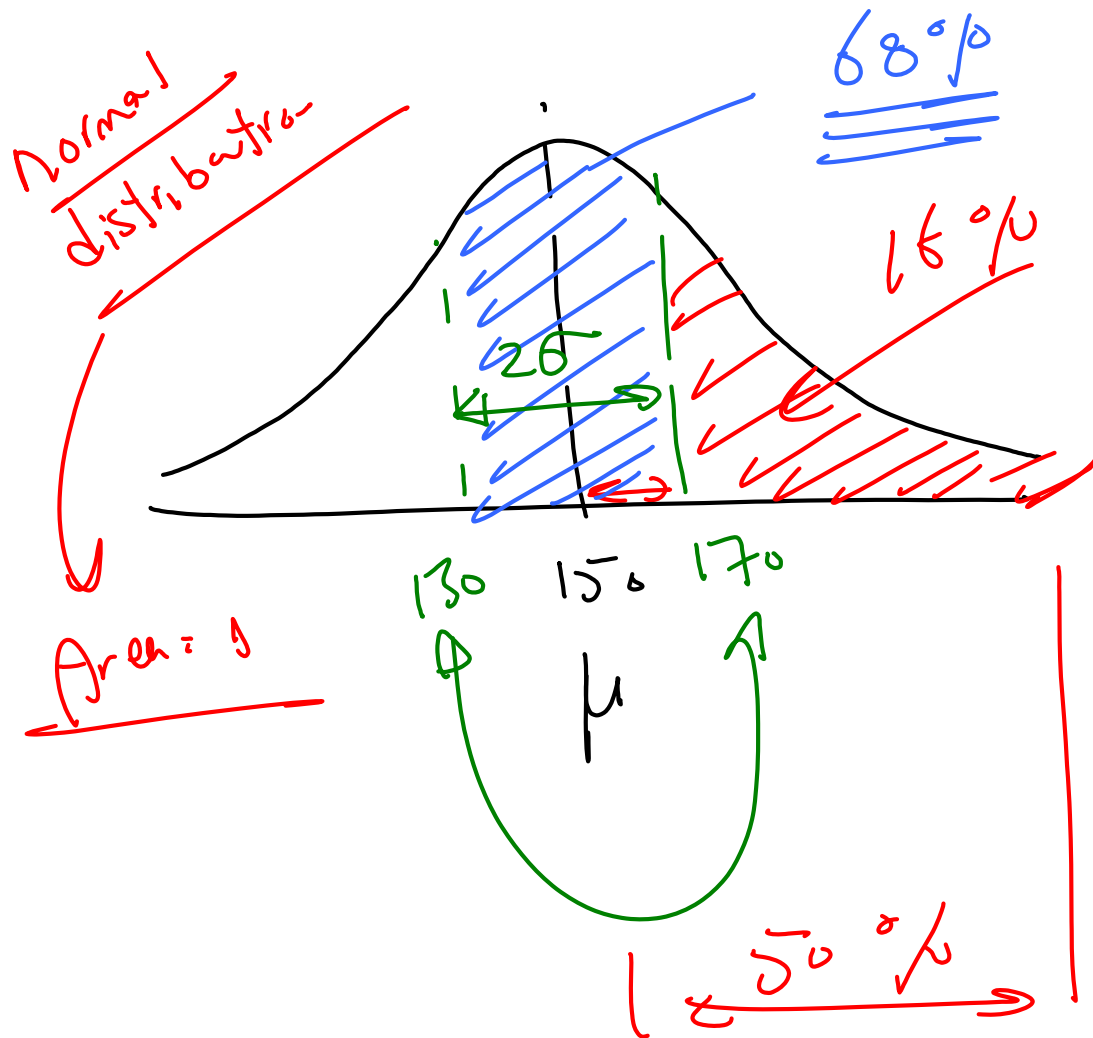
$$P(X = 45) = 0$$

$$P(X > 45)$$

$$P(X \geq 45)$$

$$= \underline{\underline{15\%}}$$





$H \Rightarrow$ Random Variable height

$$P(H > 170)$$

$$\mu = 150$$

$$\sigma = 20$$

Z-table $\Rightarrow$ Coming



| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art
and science of learning from data (Fourth Edi).
- Khan Academy