Probability

Probability Distribution Function (PDF)

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

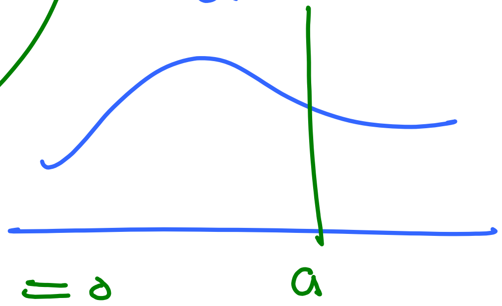
probability distribution

x	$P(X=x)$

$\rightarrow$ Continuous Random Variable $\Rightarrow$ interval

$$P(a \leq X < b) = \int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx$$



$$P(X=a) = 0$$

Probability Density Function (PDF)

- The proportion of the population whose values of X lie between a and b is given by

pdf

$$P(a < X < b) = \int_a^b f(x)\,dx$$

$\leq \quad <$

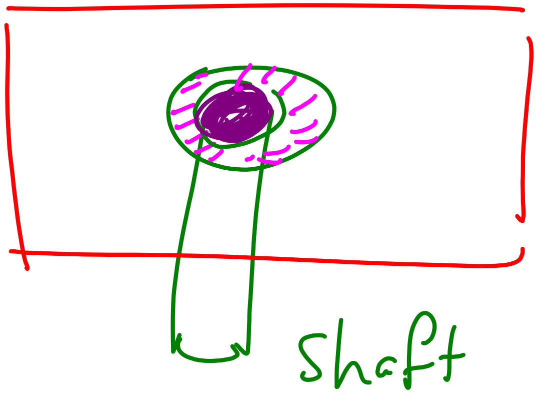
$< \quad \leq$

$\leq \quad \leq$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Example

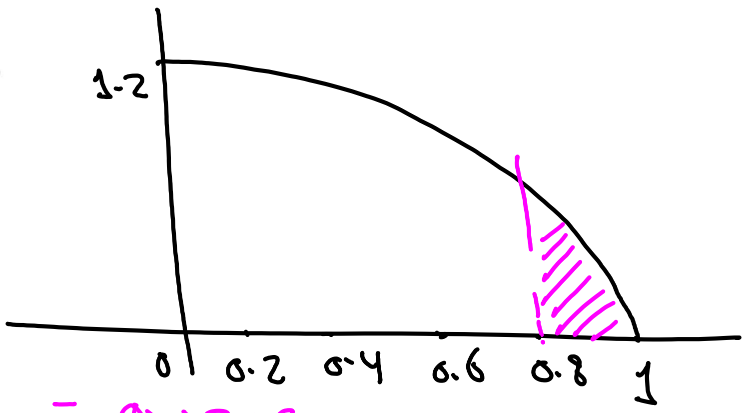
- A **hole is drilled** in a sheet-metal component, and then a **shaft is inserted** through the hole. The **shaft clearance** is equal to the difference between the radius of the hole and the radius of the shaft. Let the random variable **X denote the clearance**, in millimeters. The **probability density function of X** is



$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

interval

- Components with clearances larger than **0.8 mm must be scrapped**. **What proportion of components are scrapped**?



$$P(X \geq 0.8) = \int_{0.8}^{1} 1.25(1 - x^4)\,dx$$

$$= 1.25\left(x - \frac{x^5}{5}\right)\Big|_{0.8}^{1} = \underline{0.0819}$$

Mean

① $p(a \leq X \leq b) = \int_a^b f(x)\,dx$ ✓

② mean = Expectan(e = Expected value

$$= \mu_x = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

Variance

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$$\mu_x = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)\,dx$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu_x^2$$

Example

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$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find the **mean clearance and the variance** of the clearance

$$\mu_x = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x\left[1.25(1-x^4)\right]dx$$

$$= 1.25\left(\frac{x^2}{2} - \frac{x^6}{6}\right)\Big|_0^1$$

$$= 0.4167$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu_x^2$$

$$= \int_0^1 x^2\left[1.25(1-x^4)\right]dx - (0.4167)^2$$

$$= 0.0645$$

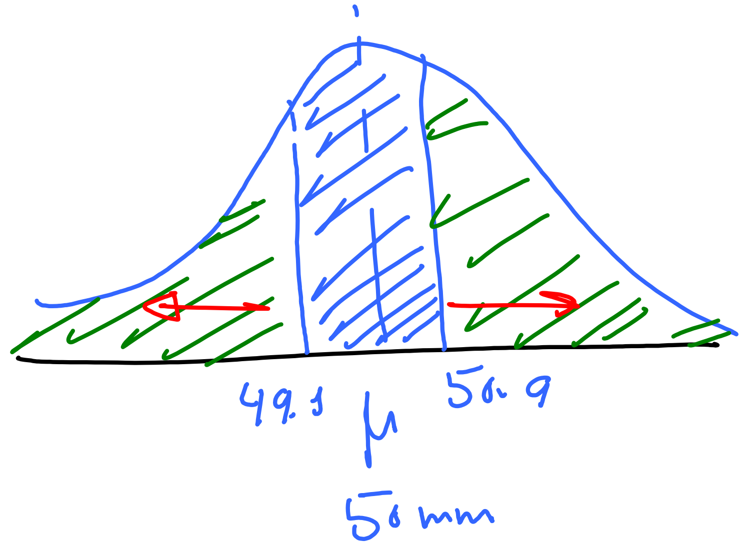
Chebyshev's Inequality

- **Chebyshev's inequality** relates the mean and the standard deviation by providing a **bound on the probability** that a random variable takes on a value that **differs from its mean by more than a given multiple of its standard deviation**.

$$P(|X-\mu_X| \geq k\sigma_X) \leq \frac{1}{k^2}$$

Example

- The length of a rivet manufactured by a certain process has mean $\mu_x = 50\ mm$ and standard deviation $\sigma_x = 0.45\ mm$.
- What is the largest possible value for the probability that the length of the rivet is outside the interval 49.1–50.9 mm?



$$P\left(|X-\mu_x| \geq k\sigma_x\right) \leq \frac{1}{k^2}$$

$$P\left(X \leq 49.1 \text{ or } X \geq 50.9\right) = P\left(|X-50| \geq 0.9\right)$$

$$P\left(|X-50| \geq 0.9\right) = P\left(|X-50| \geq 2\sigma_x\right) \leq \frac{1}{4}$$

Example

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy