Probability

Bernoulli Distribution

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Probability Density Function (PDF)

- **Statistical inference** involves **drawing a sample from a population** and **analyzing** the sample data to learn about the population.
- In many situations, **one has an approximate knowledge** of the probability mass function or probability density function of the **population**.
  - **Approximated by one of several standard families**
  - **Describe some of these standard functions**,
  - Describe **some conditions under which** it is appropriate

Continuous Random Variable

pdf

pmf

discrete Random Variable

Bernoulli Distribution

- Imagine an experiment that can **result in one of two outcomes**. One outcome is labeled **success**, and the other outcome is labeled **failure**.
  - Toss of a coin
  - accept / reject component

↳ defective

Fail — Succeed

function → Not function

low

high

$p \Rightarrow$ probability of success

$1-p \Rightarrow$ probability of failure

Bernoulli trial

Bernoulli Distribution

- For any Bernoulli trial, we define a **random variable** X as follows: If the experiment results in **success**, then **X = 1**. Otherwise, **X = 0**.

failure

$X = 1 \Rightarrow$ Success

$X = 0 \Rightarrow$ failure

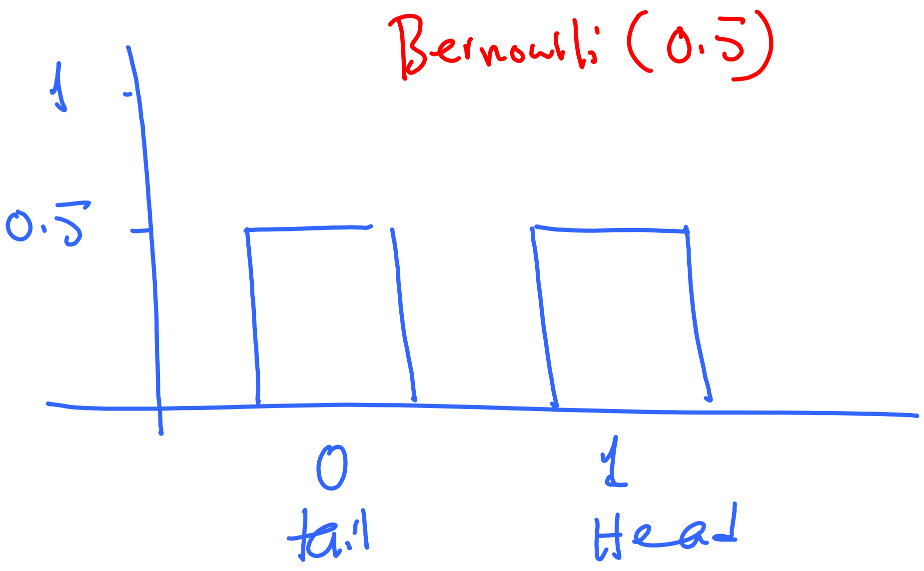
$P(0) = P(X = 0) = 1 - p$

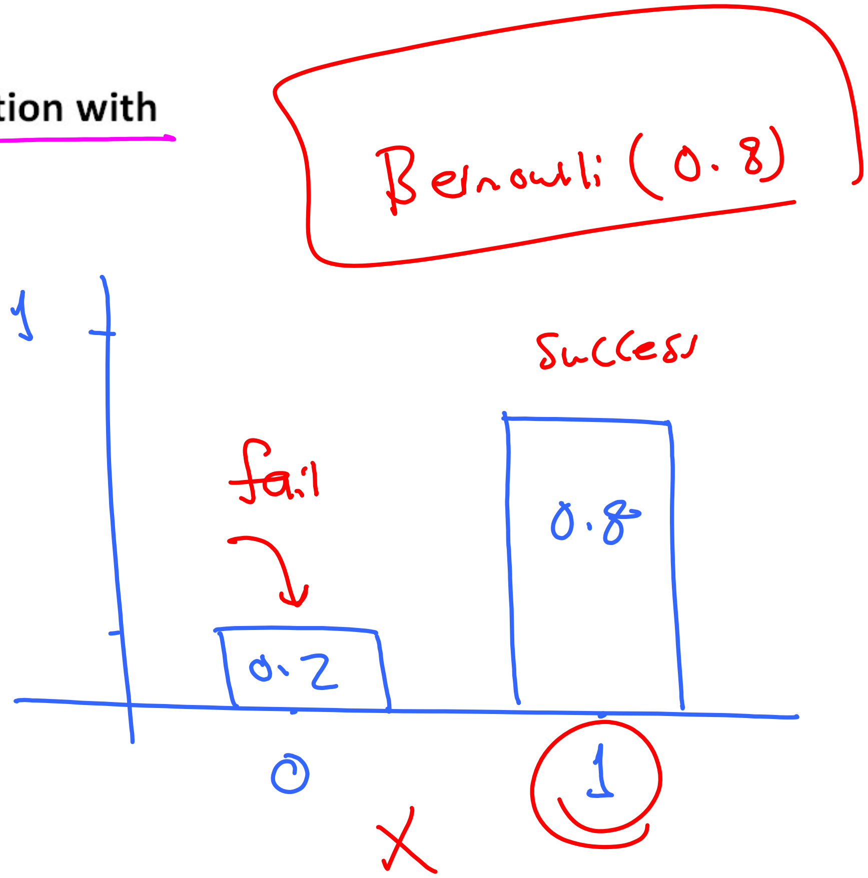
$P(1) = P(X = 1) = p$

Success

Bernoulli Distribution

- The random variable X is said to have the **Bernoulli distribution with parameter p**.
- The notation is **X ~ Bernoulli(p)**





Bernoulli Distribution

- Consider whether the next person buying a computer at a university bookstore **buys a laptop or a desktop model**. Let

$$X = \begin{cases} 1 & \text{if the customer purchases a laptop computer} \\ 0 & \text{if the customer purchases a desktop computer} \end{cases}$$

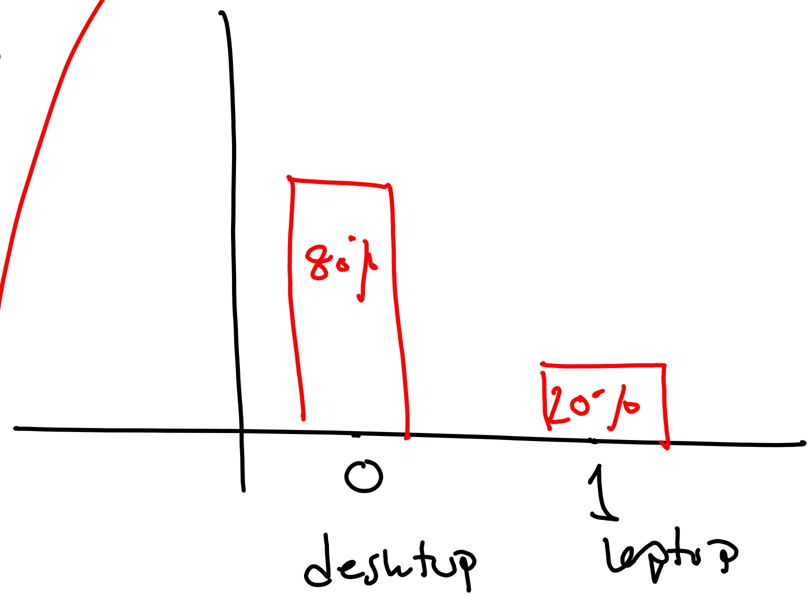
- If **20%** of all purchasers during that week select a laptop,

$X = 1 \Rightarrow$ Success $\Rightarrow$ laptop

$X = 0 \Rightarrow$ failure $\Rightarrow$ desktop

$P(0) = P(X=0) = 0.8$

$P(1) = P(X=1)$
$= 0.2$



Bernoulli Distribution

- Consider whether the next person buying a computer at a university bookstore buys a laptop or a desktop model.

$$P(x) = \begin{cases} 0.8 & x = 0 \\ 0.2 & x = 1 \\ 0 & x \neq 0 \text{ or } 1 \end{cases}$$

Example - 01

- A **coin** has **probability 0.5 of landing heads when tossed**. Let X = 1 if the coin comes up heads, and X = 0 if the coin comes up tails. What is the distribution of X?

$$\left.\begin{array}{l} P(X=1) = P(1) = 0.5 \\ P(X=0) = P(0) = 0.5 \end{array}\right\} \boxed{X \sim \text{Bernoulli}(0.5)}$$

Example - 02

- A **die** has probability **1/6 of coming up 6 when rolled**. Let X = 1 if the die comes up 6, and X = 0 otherwise. What is the distribution of X?

$p = P(X=1) = P(1) = 1/6$

Bernoulli $(1/6)$

Example - 03

10 %

- **Ten percent** of the components manufactured by a certain process are **defective**. A component is chosen at random. Let **X = 1** if the component is **defective**, and X = 0 otherwise. What is the distribution of X?

defective

Success

$$P(\text{defective}) = P(\text{success}) = P(X=1) = 0.1$$

$$P(0) = 0.9 \longrightarrow X \sim \text{Bernoulli}(0.1)$$

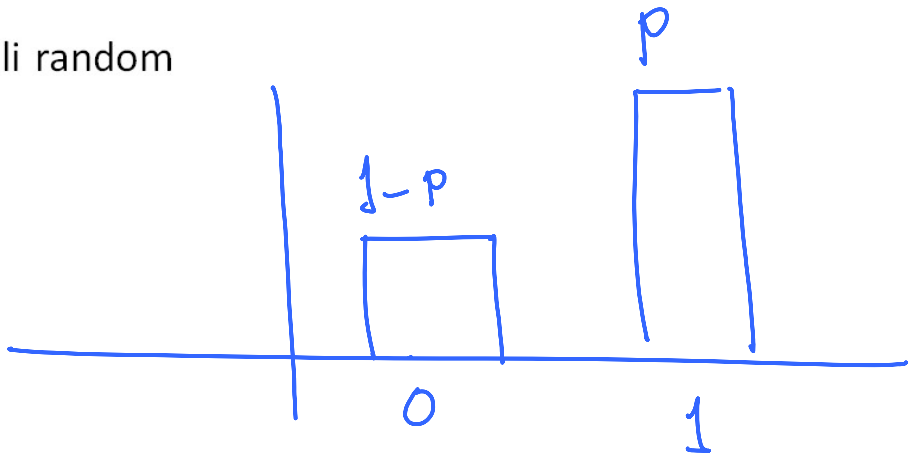
Mean and Variance

- It is easy to compute the mean and variance of a Bernoulli random variable. If $X \sim Bernoulli(p)$, then

Mean = Expected Value

$$\mu_X = \sum_x x\, p(x)$$

$$= 0 * (1-p) + 1\,(p) = p$$

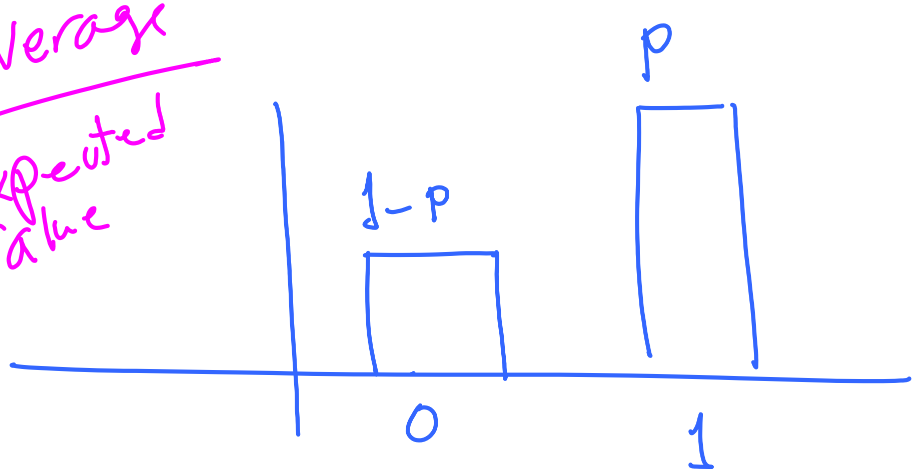


Expected Value

$$\mu_x = E(x) = p$$

Average

Expected Value



$$\sigma_x^2 = \sum_x (x-\mu)^2 p(x)$$

$$\sigma_x^2 = \sum_x x^2 p(x) - \mu^2$$

$$= \cancel{(0^2)(1-p)} + (1)(p) - p^2 = p - p^2 = p(1-p)$$

Variance

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy