

Probability

Binomial Distribution

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| Introduction

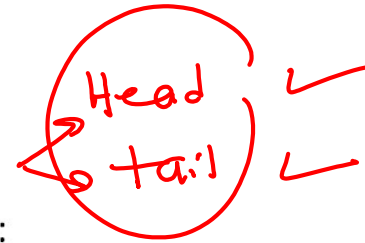
In many applications, each observation is binary: It has one of two possible outcomes. For instance, a person may

- **Accept**, or **decline**, an offer from a bank for a credit card,
- Have, or not have, **health insurance**,
- **Vote yes or no** in a referendum

| Introduction

Consider the following random experiments and random variables:

- Flip a coin 10 times. $X = \text{Number of Heads obtained}$
- A worn machine tool produces 1 % defective parts.
- Each sample of air has a 10% chance of containing a particular rare molecule.
- Of all bits transmitted through a digital transmission channel, 10% are received in error.



99 %
1 %

$$X = \frac{\# \text{ air sample}}{18 \text{ sample}}$$

| Introduction

Consider the following random experiments and random variables:

- A multiple-choice test contains 10 questions. = # 4 choice
- In the next 20 births at a hospital
- Of all patients suffering a particular illness, 35% experience improvement from a particular medication. =

$X = \#$ questions answered correctly

male
female

$X = \#$ female births

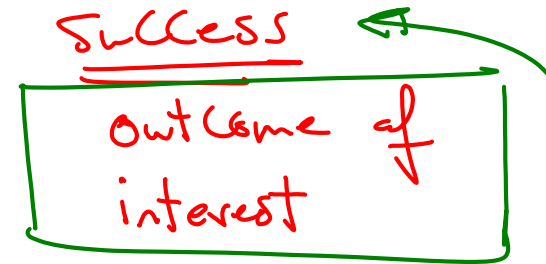
| Introduction

- Sampling a single component from a lot and determining whether it is defective is an example of a Bernoulli trial.

Several independent
Bernoulli trial $\Rightarrow$ binomial
distribution

| Conditions for Binomial Distribution

- Each of n trials has two possible outcomes.
- Each trial has the same probability of a success.
- The n trials are independent.



failure $\Rightarrow 1-p$

$X \Rightarrow$ binomial Random variable $\Rightarrow$ # successes in n trial

n , p

$$\Rightarrow X \sim \text{Bin}(n, p)$$

| Flipping a coin n times

- Each trial is a flip of the coin. There are **two possible outcomes** for each flip, head or tail. Let's identify (arbitrarily) head as a success.
- If head or tail are equally likely, the probability of head (a success) equals $p = 0.50$. This **success probability is the same for each flip**.
- The flips are independent, since the result for any specific flip does not depend on the outcomes of previous flips.

→ head

$$P(H) = \underline{0.5}$$

Example - 01

- A biased coin has probability 0.6 of coming up heads. The coin is tossed three times. Let X be the number of heads. Then $X \sim \text{Bin}(3, 0.6)$. We will compute $P(X = 2)$

$n \Rightarrow \# \text{ trial} \Rightarrow 3$
 $p \Rightarrow 0.6 \rightarrow \text{head}$

T	T	T
T	T	H
T	H	T
T	H	H
H	T	T
H	T	H
H	H	T
H	H	H

Handwritten annotations: Red circles around the outcomes THH, HTH, and HHT. Red boxes around the rows (T, H, H), (H, T, H), and (H, H, T). Arrows point from these circles and boxes to the probability calculation on the right.

$$P(HHT) = P(H) \cdot P(H) \cdot P(T) \\ = (0.6)^2 (0.4)^1$$

$$P(X=2) = P(HHT \text{ or } HTH \text{ or } THH) \\ = P(HHT) + P(HTH) + P(THH)$$

Example - 01

$$P(X=2) = \underline{3} (\underline{0.6})^2 (\underline{0.4})^1$$

ways to choose r

Success probability (p)

failure probability (1-p)

from n

$\Downarrow$

$\boxed{nCr}$

$$\binom{n}{r} = \frac{n!}{r! (n-r)!}$$

| Binomial Distribution PMF

- If $X \sim \text{Bin}(n, p)$, the probability mass function of X is

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

trials

Success
probability

probability
of failure

probability
of success

nC_x

binomial
Coefficient

$$\binom{n}{x} = \frac{n!}{x! (n-x)!}$$

| References

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Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy