Probability

Binomial Distribution

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

Introduction

In many applications, each observation is **binary**: It has one of two possible outcomes. For instance, a person may

- **Accept**, or **decline**, an offer from a bank for a credit card,
- Have, or not have, **health insurance**,
- **Vote yes or no** in a referendum

Introduction

Head

tail

Consider the following random experiments and random variables:

- Flip a coin 10 times. X = Number of Heads obtained
- A worn machine tool produces 1 % defective parts.
- Each sample of air has a 10% chance of containing a particular rare molecule.
- Of all bits transmitted through a digital transmission channel, 10% are received in error.

99 %

1 %

X = # air sample

18 Sample

Introduction

Consider the following random experiments and random variables:

- A multiple-choice test contains 10 questions. = # 4 choice
- In the next 20 births at a hospital
- Of all patients suffering a particular illness, 35% experience improvement from a particular medication.

X = # questions answered correctly

male

female

X = # female births

Introduction

- **Sampling a single component** from a lot and determining whether it is **defective** is an example of a **Bernoulli trial**.

Several Independent Bernoulli Trial $\Longrightarrow$ binomial distribution

Conditions for Binomial Distribution

- Each of n trials has two possible outcomes.
- Each trial has the same probability of a success.
- The n trials are independent.

success

$\boxed{\text{outcome of interest}}$

p

failure ⇒ 1-p

X ⇒ binomial Random variable ⇒ # successes in n trial

⇒ n, p ⇒ $\boxed{X \sim Bin(n, p)}$

Flipping a coin n times

- Each trial is a flip of the coin. There are **two possible outcomes** for each flip, head or tail. Let's identify (**arbitrarily**) **head as a success**.

head

- If head or tail are **equally likely**, the probability of head (a success) equals $p = 0.50$. This **success probability is the same for each flip**.

$P(H) = 0.5$

- The **flips are independent**, since the result for any specific flip does not depend on the outcomes of previous flips.

Example - 01

- A **biased** coin has probability **0.6** of coming up **heads**. The coin is tossed **three times**. Let X be the number of heads. Then $\boldsymbol{X} \sim \boldsymbol{Bin}(\mathbf{3}, \mathbf{0.6})$. We will compute $P(X = 2)$

$n \Rightarrow$ # trial $\Rightarrow 3$

$p \Rightarrow 0.6 \rightarrow$ head

T T T
T T H
T H T
$\boxed{\text{T H H}}$
H T T
$\boxed{\text{H T H}}$
$\boxed{\text{H H T}}$
H H H

THH

HTH

HHT

$$P(HHT) = P(H) \cdot P(H) \cdot P(T)$$
$$= (0.6)^2 (0.4)^1$$

$$P(X=2) = P(HHT \text{ or } HTH \text{ or } THH)$$
$$= P(HHT) + P(HTH) + P(THH)$$

Example - 01

$$P(X=2) = \underline{\underline{3}}\,(\underline{\underline{0.6}})^2\,(\underline{\underline{0.4}})^1$$

failure probability $(1-p)$

success probability (p)

ways to choose $\underline{\underline{r}}$ from n

by

$$\boxed{{}^nC_r} \qquad \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Binomial Distribution PMF

- If $X \sim \mathbf{Bin}(n, p)$, the probability mass function of X is

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

probability of failure

\# trials

Success probability

nC_x

binomial coefficient

probability of success

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy