Probability

Binomial Distribution – Solved Examples

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Conditions for Binomial Distribution

- **Each of n trials has two possible outcomes**. The **outcome of interest** is called a **success** and the other outcome is called a failure.
- **Each trial has the same probability of a success**. This is denoted by p, so the probability of a success is $\boldsymbol{p}$ and the probability of a failure is $1 - p$.
- The **n trials are independent**. That is, the result for one trial does not depend on the results of other trials.

The binomial random variable **X is the number of successes in the n trials**

X has the binomial distribution with parameters n and p, denoted $\boldsymbol{X \sim Bin(n, p)}$.

Binomial Distribution PMF

- If $\boldsymbol{X} \sim \textbf{Bin}(\boldsymbol{n}, \boldsymbol{p})$, the probability mass function of $\boldsymbol{X}$ is

$$\boldsymbol{p(x) = \binom{n}{x} p^{x}(1-p)^{n-x}}$$

- $\boldsymbol{n}$: # independent Bernoulli Trials
- $\boldsymbol{p}$: Success probability
- $\binom{\boldsymbol{n}}{\boldsymbol{x}} = \frac{\boldsymbol{n}!}{\boldsymbol{x}!(\boldsymbol{n}-\boldsymbol{x})!}$ binomial coefficient = nCx

Example - 01

- A **lot** contains several thousand components, **10%** of which are **defective**. **Seven** components are sampled from the lot. Let **X** represent the **number of defective components** in the sample. **What is the distribution of X?**

defective → Outcome of interest ⇒ Success

$p = 0.1$

$n = 7$

$$\boxed{X \sim Bin(7, 0.1)}$$

Example - 02

pmf

- Find the **probability mass function** of the random variable X if X ~ Bin(10, 0.4). Find P(X = 5).

n p

$$5! = 5\times4\times3\times2\times1$$

$$6! = 6\times\underline{5\times4\times3\times2\times1}$$

$$8! = 8\times7\times6\times\underline{\underline{5!}}$$

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

$$P(5) = \binom{10}{5} (0.4)^{5} (0.6)^{5} = \frac{10!}{5!\,(10-5)!} (0.4)^{5} (0.6)^{5}$$

$$= 0.2007$$

Example - 03

- A fair die is rolled eight times. Find the probability that no more than 2 sixes come up

$\frac{1}{6}$ $\quad 8$

Outcome of interest → six

$p \longrightarrow 1/6$

$n \longrightarrow 8$

$$P(x) = \binom{8}{x} \left(\frac{1}{6}\right)^{x} \left(5/6\right)^{n-x}$$

$$X \sim Bin(8, 1/6)$$

$$\boxed{P(X \leq 2)} = P(X=0 \text{ or } X=1 \text{ or } X=2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

Cumulative binomial distribution ⇒ table

$$F(x) = P(X \leq x)$$

Cumulative binomial distribution

$$F(x) = P(X \leq x) = \sum_{k=0}^{x} \frac{n!}{k!\,(n-k)!}\, p^{k}\, (1-p)^{(n-k)}$$

\# n

p

x

$\Rightarrow$ $n = 8$

$\Rightarrow$ $x = 2$

$\Rightarrow$ $p = 1/6$

Example - 04

- A large industrial firm allows a discount on any invoice that is paid within 30 days. Of all invoices, **10%** receive the discount. In a company audit, **12 invoices** are sampled at random. What is the probability that fewer than 4 of the 12 sampled invoices receive the discount?

Outcome of interest ⇒ discount

$p = 0.1$

$n = 12$

table

$\boxed{P(X < 4)} = P(0) + P(1) + P(2) + P(3)$

$P(0) = \binom{12}{0}(0.1)^0(0.9)^{12}$

$P(1) = \binom{12}{1}(0.1)^1(0.9)^{11}$

$\downarrow$

$P(3)$

$n = 12$
$x = 3$
$p = 0.1$

$\boxed{0.974}$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy