Probability

Binomial Distribution – Mean and Variance

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Bernoulli Distribution

$$\mu_x = p$$

$$\sigma_x^2 = p(1-p) \Rightarrow \sigma_x = \sqrt{p(1-p)}$$

The Mean

mean

- If a **fair coin** is tossed **10 times**, we expect on the **average** to see **five heads**.
- The number **5 comes from** multiplying the success probability (0.5) by the number of trials (10).
- This method works in general. If we perform ***n* Bernoulli trials**, each with **success** probability ***p***, the **mean** number of successes is ***np***

$\boxed{\mu_x = np}$

\# trial

Success probability

$X \sim Bin(n, p) \Rightarrow \mu_x = np$

$X \Rightarrow$ Sum of n Bernoulli Variables

mean $= p$

The Variance

- We can compute σ_X^2 by again noting that X is the sum of independent Bernoulli random variables and recalling that the variance of a Bernoulli random variable is $p(1 - p)$.

Bernoulli $\Rightarrow$ $\sigma_X^2 = p(1-p)$

Binomial $\Rightarrow$ $np(1-p)$

To conclude...

Summary

If $X \sim \text{Bin}(n, p)$, then the mean and variance of X are given by

$$\mu_X = np \tag{4.5}$$

$$\sigma_X^2 = np(1-p) \tag{4.6}$$

mean

Varian6

trials

success probability

$$\sigma_X = \sqrt{np(1-p)}$$

Example - 01

outcome of interest

A general contracting firm experiences **cost overruns** on **20%** of its contracts. In a company audit, **20 contracts** are sampled at random.

- Find the **mean number** that experience cost overruns.
- Find the **standard deviation** of the number that experience cost overruns.

$$\mu_x = np = 20 * 0.2 = 4$$

$$\sigma_x^2 = np(1-p) = 20 * 0.2(0.8) = 3.2$$

$$\sigma_x = \sqrt{3.2}$$

When the number of trials n is large…

- It can be **tedious** to calculate binomial probabilities of all the possible outcomes. Often, it's adequate merely to **use the mean and standard deviation** to describe where most of the probability falls
- The **binomial distribution has a bell shape** when n **is large**, so in that case, we can **use the normal distribution** to approximate the binomial distribution and conclude that nearly **all the probability falls between $\mu - 3\sigma$ and $\mu + 3\sigma$**

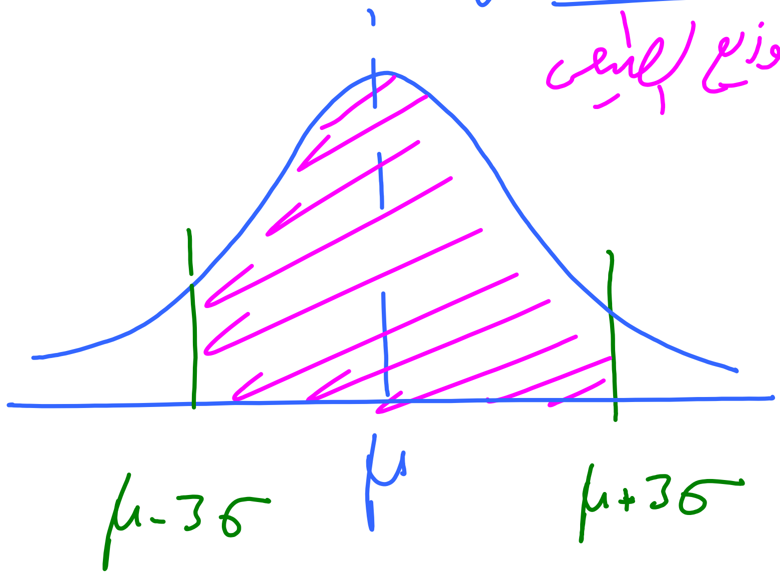
μ

σ_x

bell shape

normal distribution

التوزيع الطبيعي



References

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