

Probability

| Basic Ideas

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| Intro...

- The development of the theory of probability was financed by seventeenth-century gamblers, who hired some of the leading mathematicians of the day to **calculate the correct odds for certain games of chance**.

| Intro...

- The term **probability** refers to the **study of randomness and uncertainty**
 - It is **likely** that the **Dow-Jones** average will increase by the end of the year
 - There will **probably** be at least one section of that course offered next year
 - It is **expected** that at least **20,000** concert tickets will be sold

| Terminology

- To make a systematic study of probability, we need some **terminology**

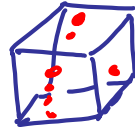
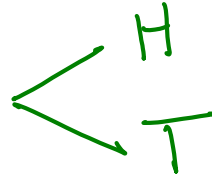
Experiment

تجربہ عسوانہ

ممکنہ ہر چیز تیار ہونے کی
سبقاً دونوں اجراء کیے

■ An **experiment** is a process that results in an outcome that cannot be predicted in advance with certainty.

- Tossing a **coin**,
- Rolling a **die**,
- Measuring the diameter of a bolt,
- **Weighing** the contents of a box of cereal,
- **Selecting a card** or cards from a deck,



$\Rightarrow \{1, 2, 3, 4, 5, 6\}$

| Trial

- Performance of an experiment is called a **trial** of the experiment

Outcome

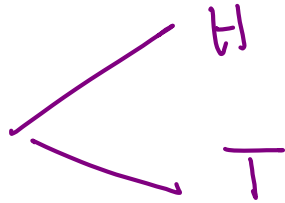
النتيجة
الخروج

- An observed result, on a trial of the experiment, is called an outcome

Sample Space

- The **set of all possible outcomes** of an experiment is called the **sample space** for the experiment.
- Denoted by S

Toss a coin



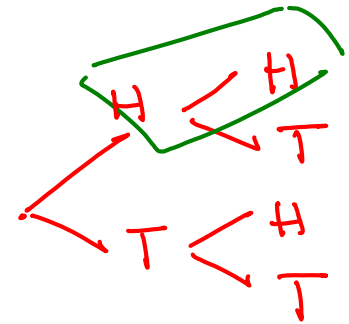
$$S = \{H, T\}$$

Rolling die

$$S = \{1, 2, 3, 4, 5, 6\}$$

فهي العينة

بأنه إبتداءً من التجربة



HH
HT
TH
TT

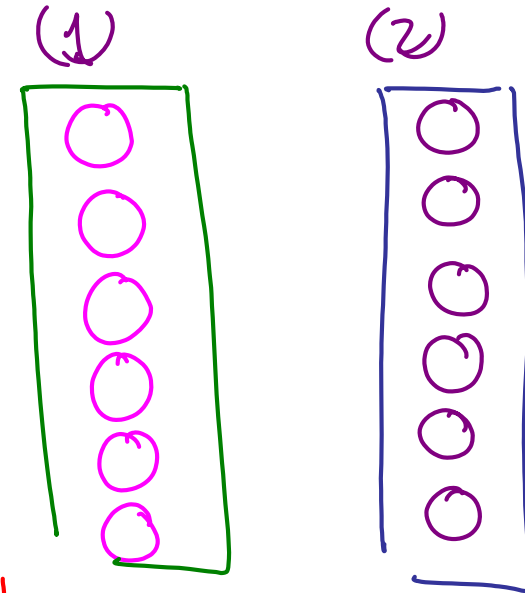
$$S = \{HH, HT, TH, TT\}$$

| Example – Finite Sample Space

- For **tossing a coin**, we can use the set **{Heads, Tails}** as the sample space.

| Example – Finite Sample Space

- Two gas stations are located at a certain intersection. Each one has six gas pumps. Consider the experiment in which the number of pumps in use at a particular time of day is determined for each of the stations. An experimental outcome specifies how many pumps are in use at the first station and how many are in use at the second one. One possible outcome is (2, 2), another is (4, 1), and yet another is (1, 4). The **49 outcomes** in S are displayed in the accompanying table



$$S = \left\{ \begin{array}{l} (0, 0), (0, 1), (0, 2), (0, 3), \dots, (0, 6), \\ (1, 0), (1, 1), (1, 2), \dots, (1, 6), \\ \vdots \\ (6, 0), (6, 1), \dots, (6, 6) \end{array} \right\} \Rightarrow \underline{\underline{49}}$$

| Example – Infinite Sample Space

- Some experiments have sample spaces with an **infinite number of outcomes**
- Punch with diameter 10 mm punches holes in sheet metal. Because of variations in the angle of the punch and slight movements in the sheet metal, the diameters of the holes vary between 10.0 and 10.2 mm
- Reasonable sample space is the **interval (10.0, 10.2)**, or in set notation, $\{x \mid 10.0 < x < 10.2\}$

↑

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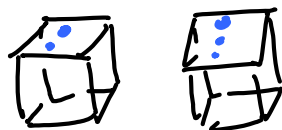
Continuous

Event

قوت چیزیه که زنده باشه

$$\text{event} = \{2, 4, 6\}$$

- In our study of probability, we will be interested not only in the individual outcomes of S but also in various collections of outcomes from S .
- A subset of a sample space is called an event
- For example, we might be interested in the probability that a **die** comes up an even number.



$$\Rightarrow \underline{\text{Sum} = 8}$$

$$\text{event} = \{(2, 6), (3, 5), (4, 4), (4, 4), (5, 3), (6, 2)\}$$

| Event

- In the hole punch example, we might be interested in the probability that a hole has a diameter less than 10.1 mm.
- This correspond to the subset $\{x \mid 10.0 < x < 10.1\}$
- A **given event is said to have occurred** if the outcome of the experiment is one of the outcomes in the event. For example, if a die comes up 2, the events $\{2, 4, 6\}$ and $\{1, 2, 3\}$ have both occurred, along with every other event that contains the outcome "2."

2
↓
event
happened

| References

- Understanding Boxplots <https://towardsdatascience.com/understanding-boxplots-5e2df7bcbd51>
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<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6th-box-whisker-plots/v/constructing-a-box-and-whisker-plot>