Probability

Binomial Distribution – Mean and Variance

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The Mean

- If a **fair coin** is tossed **10 times**, we expect on the **average** to see **five heads**.
- The number **5 comes from** multiplying the success probability (0.5) by the number of trials (10).
- This method works in general. If we perform $\boldsymbol{n}$ **Bernoulli trials**, each with **success** probability $\boldsymbol{p}$, the **mean** number of successes is $\boldsymbol{np}$

The Variance

- We can compute σ_X^2 by again noting that X is the sum of independent Bernoulli random variables and recalling that the variance of a Bernoulli random variable is $p(1 - p)$.

To conclude...

Summary

If $X \sim \text{Bin}(n, p)$, then the mean and variance of X are given by

$$\mu_X = np \tag{4.5}$$

$$\sigma_X^2 = np(1 - p) \tag{4.6}$$

Example - 01

A general contracting firm experiences **cost overruns** on **20%** of its contracts. In a company audit, **20 contracts** are sampled at random.

- Find the **mean number** that experience cost overruns.
- Find the **standard deviation** of the number that experience cost overruns.

When the number of trials n is large…

- It can be **tedious** to calculate binomial probabilities of all the possible outcomes. Often, it's adequate merely to **use the mean and standard deviation** to describe where most of the probability falls
- The **binomial distribution has a bell shape** when n **is large**, so in that case, we can **use the normal distribution** to approximate the binomial distribution and conclude that nearly **all the probability falls between $\mu - 3\sigma$ and $\mu + 3\sigma$**

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy