

Probability

Binomial Distribution – Solved Examples

Prof. Dr. Mostafa Elhosseini

<https://youtube.com/drmelhosseini>

| Conditions for Binomial Distribution

- **Each of n trials has two possible outcomes.** The **outcome of interest** is called a **success** and the other outcome is called a failure.
- **Each trial has the same probability of a success.** This is denoted by p , so the probability of a success is p and the probability of a failure is $1 - p$.
- **The n trials are independent.** That is, the result for one trial does not depend on the results of other trials.

The binomial random variable **X is the number of successes in the n trials**

X has the binomial distribution with parameters n and p , denoted $X \sim \text{Bin}(n, p)$.

| Binomial Distribution PMF

- If $X \sim \mathbf{Bin}(n, p)$, the probability mass function of X is

$$p(x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

- n : # independent Bernoulli Trials
- p : Success probability
- $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ binomial coefficient = nCx

| Example - 01

- A **lot** contains several thousand components, **10%** of which are **defective**. **Seven** components are sampled from the lot. Let **X** represent the **number of defective components** in the sample. **What is the distribution of X ?**

| Example - 02

- Find the **probability mass function** of the random variable X if $X \sim \text{Bin}(10, 0.4)$. Find $P(X = 5)$.

| Example - 03

- A fair die is rolled eight times. Find the probability that no more than 2 sixes come up

| Example - 04

- A large industrial firm allows a discount on any invoice that is paid within 30 days. Of all invoices, **10%** receive the discount. In a company audit, **12 invoices** are sampled at random. What is the probability that fewer than 4 of the 12 sampled invoices receive the discount?

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy