Probability

Binomial Distribution

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Introduction

In many applications, each observation is **binary**: It has one of two possible outcomes. For instance, a person may

- **Accept**, or **decline**, an offer from a bank for a credit card,
- Have, or not have, **health insurance**,
- **Vote yes or no** in a referendum

Introduction

Consider the following random experiments and random variables:

- Flip a coin 10 times.
- A worn machine tool produces 1 % defective parts.
- Each sample of air has a 10% chance of containing a particular rare molecule.
- Of all bits transmitted through a digital transmission channel, 10% are received in error.

Introduction

Consider the following random experiments and random variables:

- A multiple-choice test contains 10 questions.
- In the next 20 births at a hospital
- Of all patients suffering a particular illness, 35% experience improvement from a particular medication.

Introduction

- **Sampling a single component** from a lot and determining whether it is **defective** is an example of a **Bernoulli trial**.

Conditions for Binomial Distribution

- Each of n trials has two possible outcomes.
- Each trial has the same probability of a success.
- The n trials are independent.

Flipping a coin n times

- Each trial is a flip of the coin. There are **two possible outcomes** for each flip, head or tail. Let’s identify (**arbitrarily**) **head as a success**.

- If head or tail are **equally likely**, the probability of head (a success) equals $p = 0.50$. This **success probability is the same for each flip**.

- The **flips are independent**, since the result for any specific flip does not depend on the outcomes of previous flips.

Example - 01

- A **biased** coin has probability **0.6** of coming up **heads**. The coin is tossed **three times**. Let X be the number of heads. Then $\boldsymbol{X} \sim \boldsymbol{Bin(3, 0.6)}$. We will compute $P(X = 2)$

Example - 01

Binomial Distribution PMF

- If $\boldsymbol{X} \sim \textbf{Bin}(\boldsymbol{n}, \boldsymbol{p})$, the probability mass function of $\boldsymbol{X}$ is

$$\boldsymbol{p}(\boldsymbol{x}) = \binom{\boldsymbol{n}}{\boldsymbol{x}} \boldsymbol{p}^{\boldsymbol{x}} (\mathbf{1} - \boldsymbol{p})^{\boldsymbol{n}-\boldsymbol{x}}$$

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy