

Probability

Conditional Probability

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| Conditional Probability

- A **sample space** contains all the possible outcomes of an experiment.
- Sometimes we obtain some **additional information** about an experiment that tells us that the **outcome comes** from a certain part of the sample space.
- A **probability that is based on a part of a sample space** is called a conditional probability

| Unconditional Probability

Length	Diameter		
	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- $P(\text{diameter OK})$?
- This probability is called the unconditional probability, since it is based on the entire sample space.

| Conditional Probability

Length	Diameter		
	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

- Now **assume that a rod is sampled**, and its **length** is measured and found to **meet the specification**. What is the probability that the diameter also meets the specification?
- **knowledge** that the **length meets the specification** reduces the **sample space** from which the rod is drawn

| Example

- **P(diameter OK | length too long)?**

Length	Diameter		
	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
Too Long	2	25	13

| Example

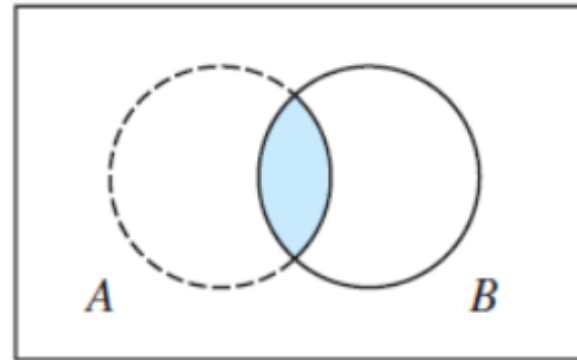
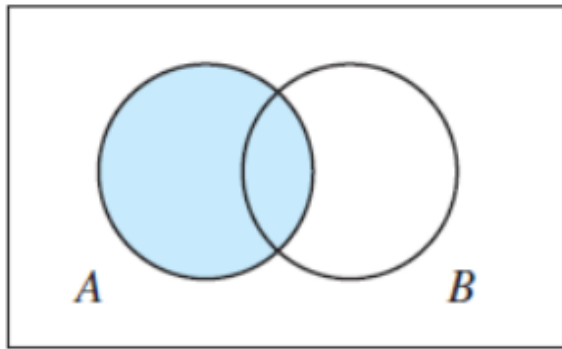
- $P(\text{diameter OK} \mid \text{length too long})?$

Length	Diameter		
	Too Thin	OK	Too Thick
Too Short	10	3	5
OK	38	900	4
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| Note that

- Divide both the numerator and denominator of this answer by the number of outcomes in the full sample space, which is 1000

| Venn Diagram



| Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the side, given that it has a flaw on top?**

| Example

- In a process that manufactures aluminum cans, the probability that a can has a flaw on its side is 0.02, the probability that a can has a flaw on the top is 0.03, and the probability that a can has a flaw on both the side and the top is 0.01. **What is the probability that a can will have a flaw on the top, given that it has a flaw on the side?**

| Independent Events

- Sometimes the **knowledge that one event has occurred does not change the probability that another event occurs.**
- In this case the **conditional and unconditional** probabilities are the **same**, and the **events are said to be independent**

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy