

Probability

Probability Distribution Function (PDF)

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| Probability Density Function (PDF)

- The proportion of the population whose values of X lie between a and b is given by

| Example

- A **hole is drilled** in a sheet-metal component, and then a **shaft is inserted** through the hole. The **shaft clearance** is equal to the difference between the radius of the hole and the radius of the shaft. Let the random variable **X denote the clearance**, in millimeters. The **probability density function of X** is

$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Components with clearances larger than **0.8 mm must be scrapped**. **What proportion of components are scrapped?**

| Mean

| Variance

| Example

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$$f(x) = \begin{cases} 1.25(1 - x^4) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find the **mean clearance and the variance** of the clearance

| Example

| Chebyshev's Inequality

- **Chebyshev's inequality** relates the mean and the standard deviation by providing a **bound on the probability** that a random variable takes on a value that **differs from its mean by more than a given multiple of its standard deviation**.

| Chebyshev's Inequality

| Example

- The length of a rivet manufactured by a certain process has mean $\mu_x = 50 \text{ mm}$ and standard deviation $\sigma_x = 0.45 \text{ mm}$.
- What is the largest possible value for the probability that the length of the rivet is outside the interval 49.1–50.9 mm?

| Example

| Example

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Agresti, A., Franklin, C. and Klingenberg, B., 2017. Statistics: The art and science of learning from data (Fourth Edi).
- Khan Academy