

Probability

Counting Methods - 01

Prof. Dr. Mostafa Elhosseini

<https://youtube.com/drmelhosseini>

| Counting

- When **computing probabilities**, it is sometimes **necessary** to **determine the number of outcomes** in a sample space

| Example

- A certain make of automobile is available in any of **three colors**: red, blue, or green, and comes with either a **large or small engine**. In **how many ways** can a buyer choose a car?

| Fundamental principle of counting

- Assume that k operations are to be performed.
- If there are n_1 ways to perform the first operation,
- and if for each of these ways there are n_2 ways to perform the second operation,
- and if for each choice of ways to perform the first two operations there are n_3 ways to perform the third operation,
- and so on,
- then the total number of ways to perform the sequence of k operations is $n_1 n_2 \cdots n_k$.

| Example 01

- When ordering a certain type of computer, there are **3** choices of hard drive, **4** choices for the amount of memory, **2** choices of video card, and **3** choices of monitor. In how many ways can a computer be ordered?

| Permutations

- A **permutation** “التبديل” is an ordering of a collection of objects. For example, there are **six permutations** of the letters **A, B, C**:

| Permutations – How to count?

- The **fundamental principle of counting** can be used to determine the number of permutations of any set of objects.
- For example, we can determine the number of permutations of a set of three objects as follows. **There are 3 choices for the object to place first.** After that choice is made, there are **2 choices remaining for the object to place second.** Then there is **1 choice left** for the object to place last.
- Therefore, the total number of ways to order three objects is **$(3)(2)(1) = 6$** .

| Example

- Five people stand in line at a movie theater. **Into how many different orders can they be arranged?**

| Note...

- Sometimes we are interested in counting the **number of permutations of subsets of a certain size** chosen from a larger set

| Example

- Five **lifeguards** “ **منقذين** ” are available for duty one Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations?

| # Permutations

- The number of permutations of k objects chosen from a group of n objects is

$$\frac{n!}{(n - k)!}$$

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy