

Probability

Discrete Random Variable [Mean and Variance]

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| Sample Mean

- The number of flaws in a 1-inch length of copper wire manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**. Let X be the **number of flaws** in a randomly selected piece of wire. Now **imagine** that we have a **sample of 100 wires**, and that the sample follows this distribution perfectly
- What is the **sample mean**?

| Population Mean

- Let X be a discrete random variable with probability mass function $p(x) = P(X = x)$.

The mean of X is given by $\mu_X = \sum_x xP(X = x)$

| Example - 01

- A certain industrial process is brought down for recalibration whenever the quality of the items produced falls below specifications. Let X represent the number of times the process is recalibrated during a week and assume that X has the following probability mass function.

Find the mean of X .

x	0	1	2	3	4
$p(x)$	0.35	0.25	0.20	0.15	0.05

| Recall: Sample variance

- Recall that for a sample $X_1, \dots, X_n$, the **sample variance** is given by

$$\sum (X_i - \bar{X})^2 / (n + 1)$$

| Population Variance

- By analogy, the **population variance of a discrete random variable X** is a weighted average of the squared differences $(x - \mu_X)^2$ where x ranges through all the possible values of the random variable X . This weighted average is computed by **multiplying each squared difference $(x - \mu_X)^2$ by the probability $P(X = x)$ and summing the results**

| Population Variance

$$\sigma_X^2 = \sum_x (x - \mu_X)^2 P(X = x)$$

$$\sigma_X^2 = \sum_x x^2 P(X = x) - \mu_X^2$$

| Example - 02

- Find the **variance and standard deviation** for the random variable X described in Example - 01, representing the number of times a process is recalibrated

x	0	1	2	3	4
$p(x)$	0.35	0.25	0.20	0.15	0.05

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy