

Probability

Normal Distribution

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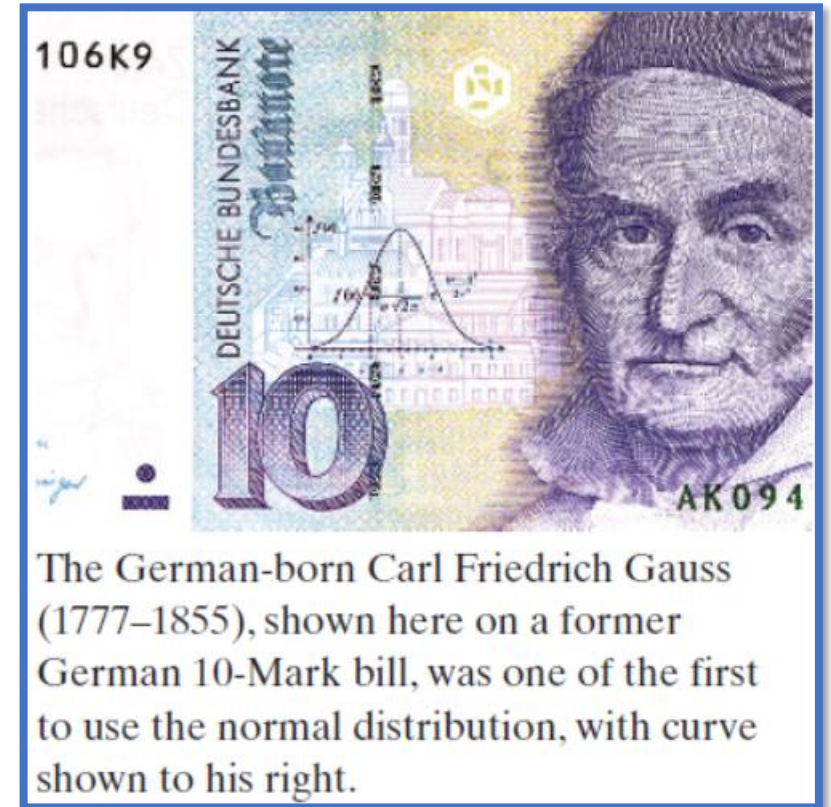
| Normal Distribution

The normal distribution (also called the **Gaussian distribution**) is by far the **most commonly used distribution** in statistics.

- Heights, weights, and other physical characteristics
- Measurement errors in scientific experiments
- **Even** when the underlying distribution is **discrete**, the **normal curve** often gives an **excellent approximation**

| Normal Distribution

- **De Moivre** presented this fundamental result, known as the **central limit theorem**, in 1733.
- Unfortunately, his work was **lost** for some time, and **Gauss** independently developed a normal distribution nearly 100 years later.
- Although De Moivre was later credited with the derivation, a **normal distribution** is also referred to as a **Gaussian distribution**



The German-born Carl Friedrich Gauss (1777–1855), shown here on a former German 10-Mark bill, was one of the first to use the normal distribution, with curve shown to his right.

| Normal Distribution

A continuous random variable X is said to have a **normal distribution** with parameters μ and σ , where $-\infty < \mu < \infty$ and $0 < \sigma$, if the *pdf* of X is

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

- The statement that X is normally distributed with parameters μ and σ^2 is often abbreviated $X \sim N(\mu, \sigma^2)$

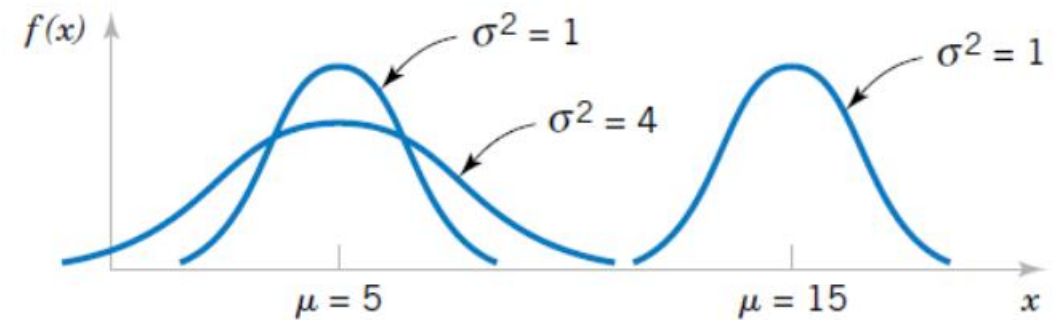
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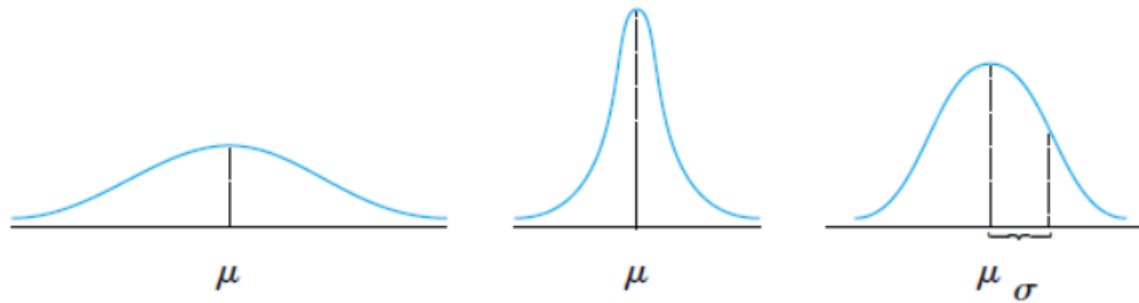
- $f(x; \mu, \sigma) \geq 0$
- $\int_{-\infty}^{\infty} f(x; \mu, \sigma) dx = 1$
- $E(X) = \mu$ and
- $V(X) = \sigma^2,$

| Normal Distribution

- Each density curve is **symmetric** about μ and **bell-shaped**, so the **center** of the bell (point of symmetry) is both the **mean** of the distribution and the **median**.
- The value of σ is the **distance from μ to the inflection points** of the curve (the points at which the curve changes from turning downward to turning upward)

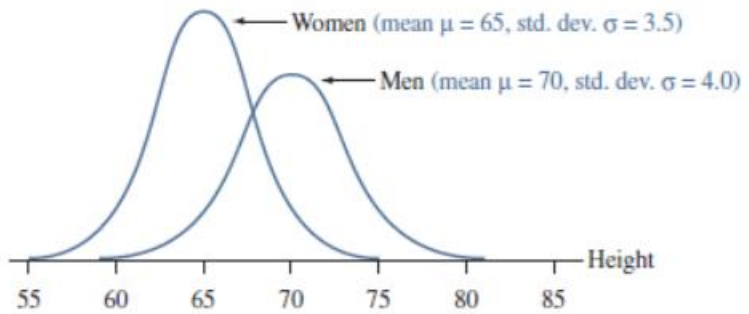


| Normal Distribution

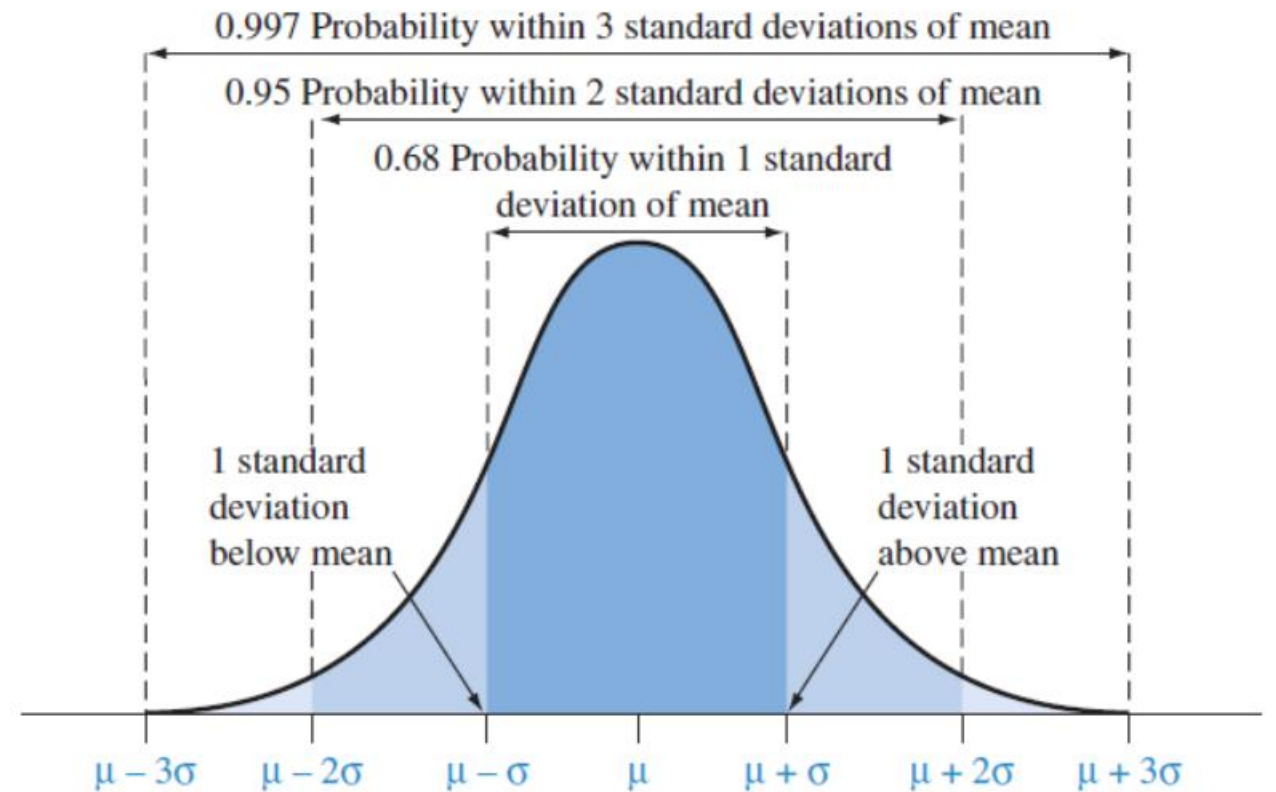


- Large values of σ yield graphs that are quite spread out about μ , whereas small values of σ yield graphs with a high peak above μ and most of the area under the graph quite close to μ .
- Thus a large σ implies that a value of X far from μ may well be observed, whereas such a value is quite unlikely when σ is small

| Normal Distribution

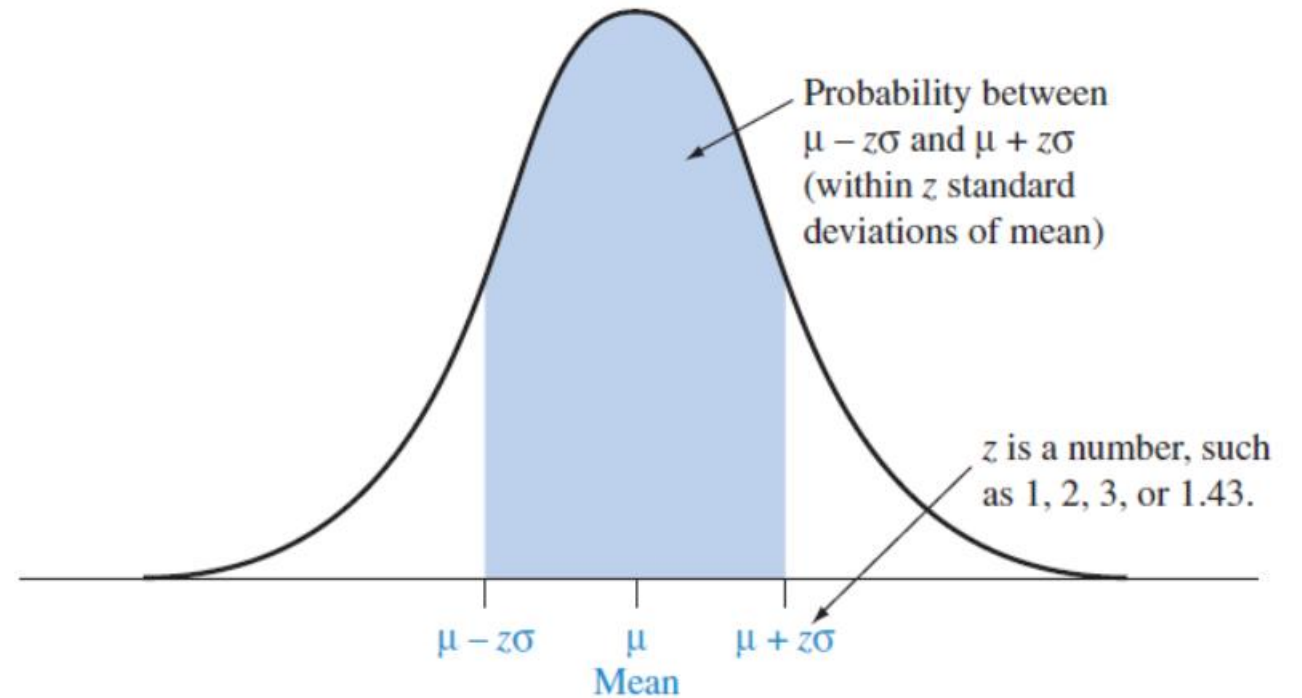


| Normal Distribution



- About 68% of the population is in the interval $\mu \pm \sigma$.
- About 95% of the population is in the interval $\mu \pm 2\sigma$.
- About 99.7% of the population is in the interval $\mu \pm 3\sigma$

| Normal Distribution



- The multiples 1, 2, and 3 of the number of standard deviations from the mean are denoted by the symbol z in general. For instance, $z = 2$ for 2 standard deviations. For each fixed number z , the probability within z standard deviations of the mean is the area under the normal curve between $\mu - z\sigma$ and $\mu + z\sigma$

| Normal Distribution

- The **proportion** of a normal population that is **within a given number of standard deviations of the mean** is the same for any normal population.
- For this reason, when dealing with normal populations, we often **convert from** the units in which the population items were originally measured **to standard units**.
- **Standard units** tell how many standard deviations an observation is from the population mean

| Normal Distribution

- To compute $P(\mathbf{a} \leq X \leq \mathbf{b})$ when X is a normal random variable with parameters μ and σ , we must determine

$$\int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

- None of the standard integration techniques can be used to evaluate the above Expression.
- Instead, for $\mu = 0$ and $\sigma = 1$, the above Expression has been calculated using numerical techniques and tabulated for certain values of $\mathbf{a}$ and $\mathbf{b}$

| Standard Normal Distribution

- The normal distribution with parameter values $\mu = 0$ and $\sigma = 1$ is called the **standard normal distribution**.
- A random variable having a standard normal distribution is called a standard normal random variable and will be denoted by **Z**.
- The pdf of Z is

$$f(z; 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad -\infty < z < \infty$$

- The **standard normal distribution** does **not** frequently **serve** as a model for a naturally arising population. Instead, it is a **reference distribution** from which information about other normal distributions can be **obtained**