Probability

| Combining Events

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

Example

- An electrical engineer has on hand **two boxes of resistors**, with **four** resistors in each box. The resistors in the first box are labeled **10 Ω** (ohms), but in fact their resistances are **9, 10, 11, and 12 Ω**. The resistors in the second box are labeled **20 Ω**, but in fact their resistances are **18, 19, 20, and 21 Ω**. The engineer chooses one resistor from each box and determines the resistance of each.
- Let **A** be the event that the **first resistor has a resistance greater than 10**, let **B** be the event that the **second resistor has a resistance less than 19**, and let **C** be the event that the **sum of the resistances is equal to 28**.
- Find a **sample space** for this experiment, and specify the subsets corresponding to the events **A, B, and C**.

Example

- **S** = {(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}

- **A** = {(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}

- **B** = {(9, 18), (10, 18), (11, 18), (12, 18)}

- **C** = {(9, 19), (10, 18)}

Combining Events

- We often construct events by combining simpler events.
- Because events are subsets of sample spaces, it is traditional to use the notation of sets to describe events constructed in this way

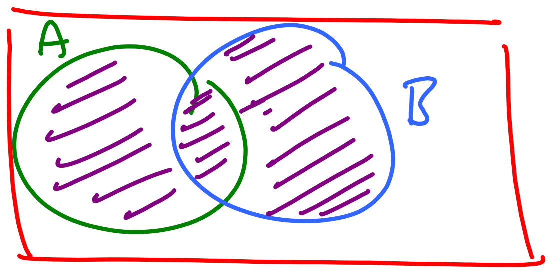
Union

- The **union of two events A and B**, denoted **A U B**, is the set of outcomes that belong either to A, to B, or to both.
- In words, A U B means "A or B." Thus the event A U B occurs whenever either A or B (or both) occurs.

$A \cup B$

$\rightarrow$ A or B

S



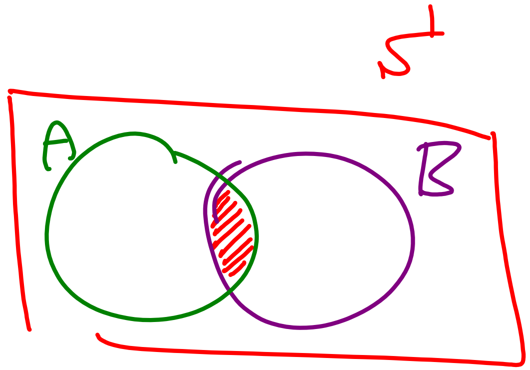
Venn diagram

Intersection

تقاطع

$A \cap B$

- The **intersection of two events A and B**, denoted **A ∩ B**, is the set of outcomes that belong both to A and to B.
- In words, A ∩ B means "A and B." Thus the event A ∩ B occurs whenever both A and B occur.



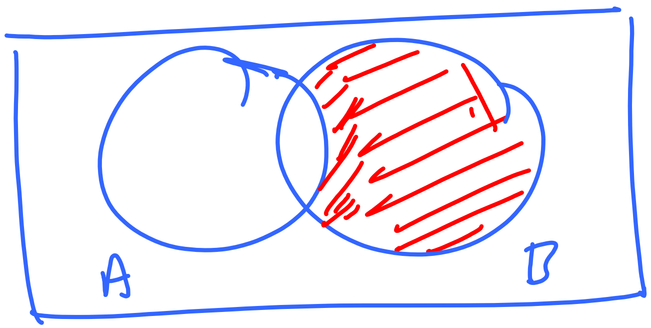
A and B

Complement

A^c

- The **complement of an event A**, denoted **A^c**, is the set of outcomes that do not belong to A. In words, A^c means "not A." Thus the event A^c occurs whenever A does not occur.

$B \cap A^c$



$B \cap A^c$

Example

- Refer to previous Example. Find B ∪ C and $A \cap B^c$.

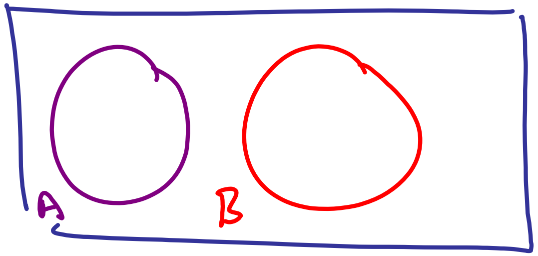
  - **S** = {(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}
  - **A** = {(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}
  - **B** = {(9, 18), (10, 18), (11, 18), (12, 18)}
  - **C** = {(9, 19), (10, 18)}

$B \cup C = \{(9, 18), (10, 18), (11, 18), (12, 18), (9, 19)\}$

$A \cap B^c = \{(11, 19), (11, 20), (11, 21), (12, 19), (12, 20), (12, 21)\}$

Mutually Exclusive Events

- The events A and B are said to be mutually exclusive if they have **no outcomes in common**.
- More generally, a collection of events **A1, A2, . . . , An** is said to be mutually exclusive if **no two of them have any outcomes in common**.



Steel pin: too long / too short

Coin: H / T

Example

- Refer to Previous Example. If the experiment is performed, is it possible for events A and B both to occur? How about B and C? A and C? Which pair of events is mutually exclusive?

  - **S** = {(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}
  - **A** = {(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)}
  - **B** = {(9, 18), (10, 18), (11, 18), (12, 18)}
  - **C** = {(9, 19), (10, 18)}