Control Engineering (1)

1st Term model Ans

Q1: $\Rightarrow$

$$\sum T = I\ddot{\theta}$$

$$\cancel{T_c} - mgL\sin\theta = I\ddot{\theta}$$

$$-mgL\sin\theta = mL^2\ddot{\theta}$$

$$L\ddot{\theta} + g\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$ $\Leftarrow$ nonlinear eqn

linearize at $\theta = 0$

$\therefore \sin\theta = \theta$ if θ is small

$$\therefore \ddot{\theta} + \frac{g}{L}\theta = 0$$

$$s^2\Theta - s\Theta(0) - \cancel{\dot{\Theta}(0)} + \frac{g}{L}\Theta = 0$$

$$\Theta\left(s^2 + \frac{g}{L}\right) = \Theta(0)\, s$$

$$\therefore \Theta(s) = \frac{\Theta_0 s}{s^2 + g/L}$$

$$\Theta(t) = \Theta_0 \cos\sqrt{g/L}$$

By MATLAB

$$\Theta(s) = \frac{\Theta_0 s^2}{s^2 + g/L} * \frac{1}{s}$$

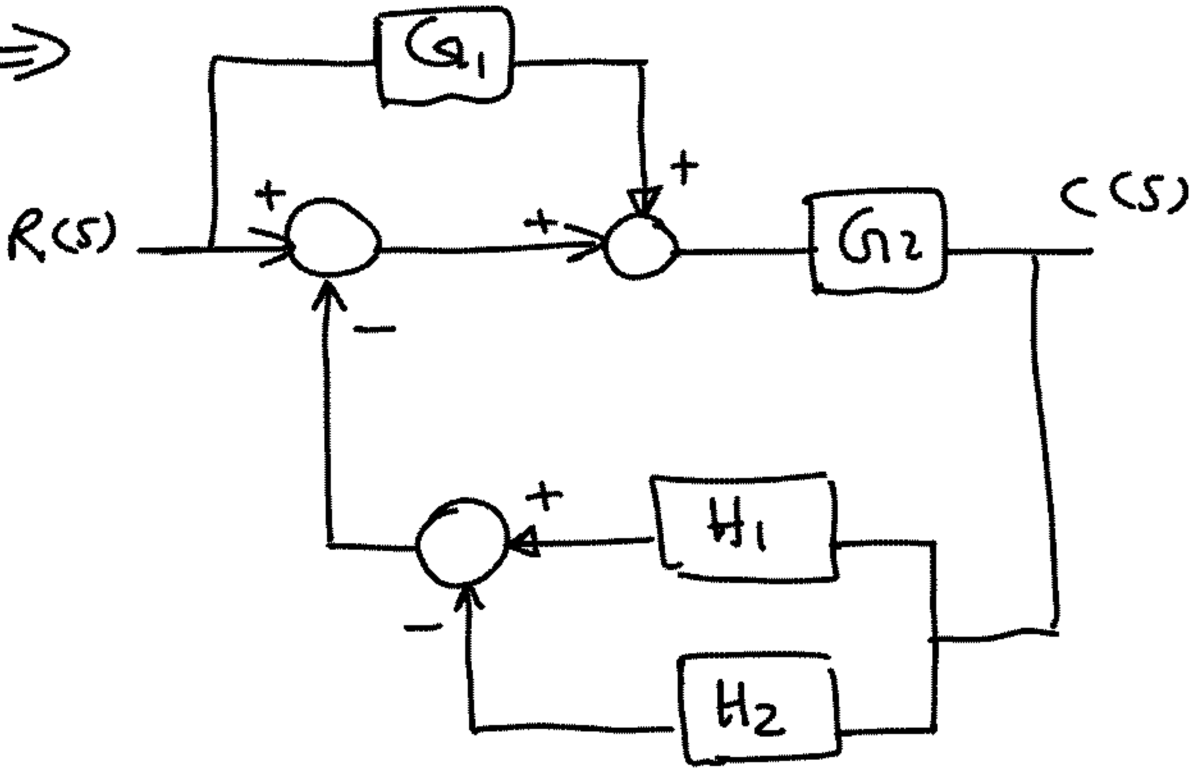
So you can use MATLAB step function as if it was

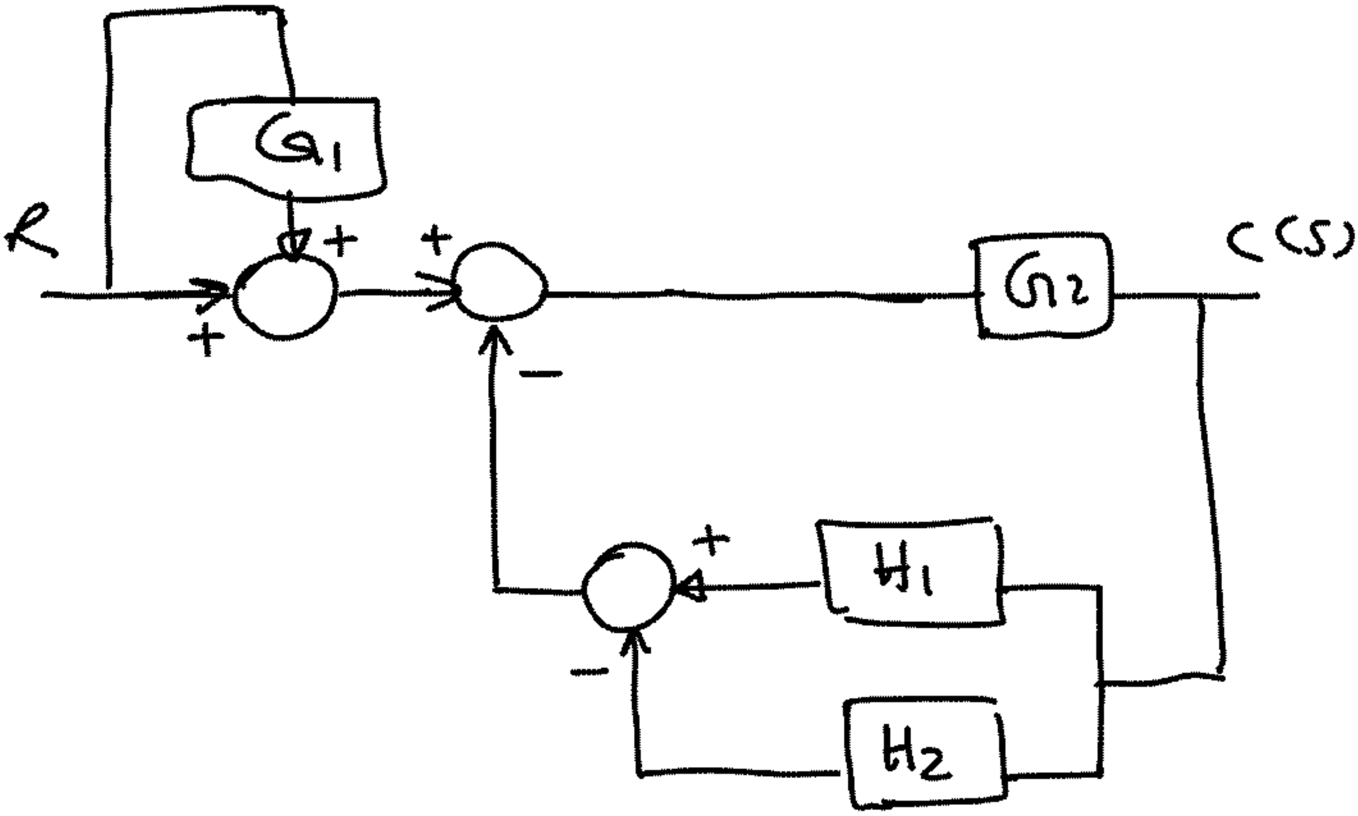
$$\frac{\Theta(s)}{M(s)} = \frac{\Theta_0 s^2}{s^2 + g/L}$$

dummy

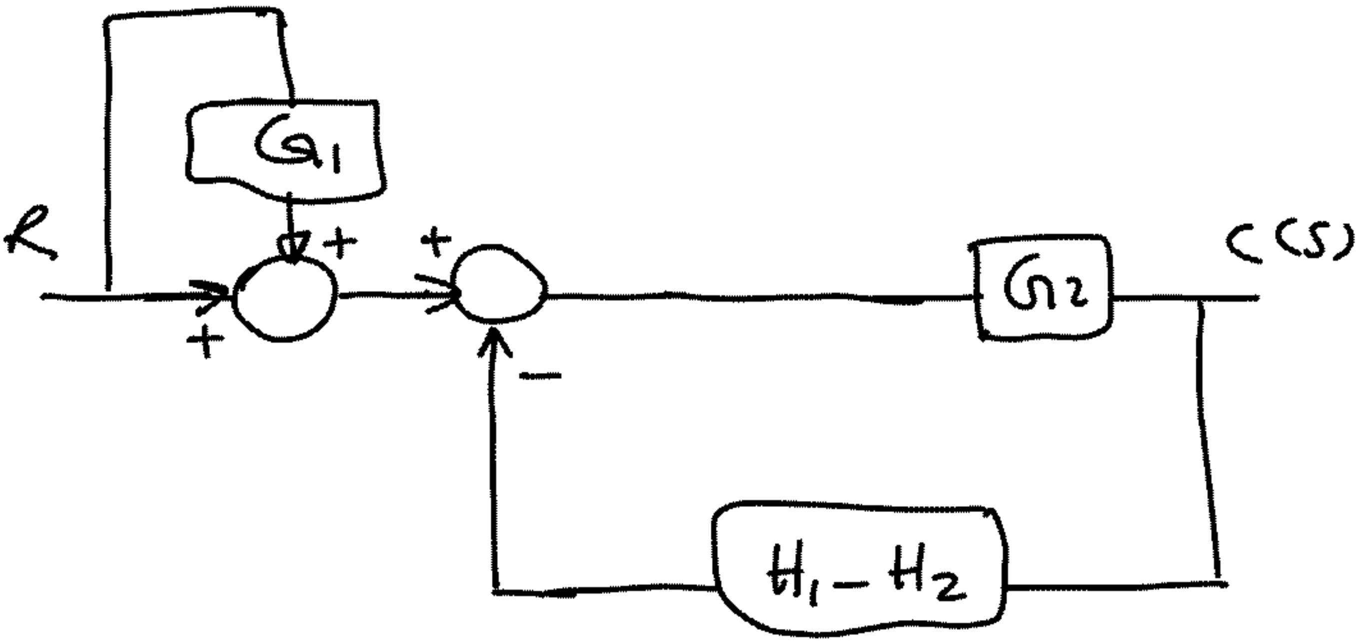
```
L = 0.5 ;
g = 9.8 ;
theta0 = 30 ;
a = [theta0 ▽ 0 ▽ 0];
b = [1 ▽ 0 ▽ g/L];
sys = tf ( a,b)
t = 0: 0.01 : 10
y = step (sys,t)
plot (t,y)
```

Q2: ⇒

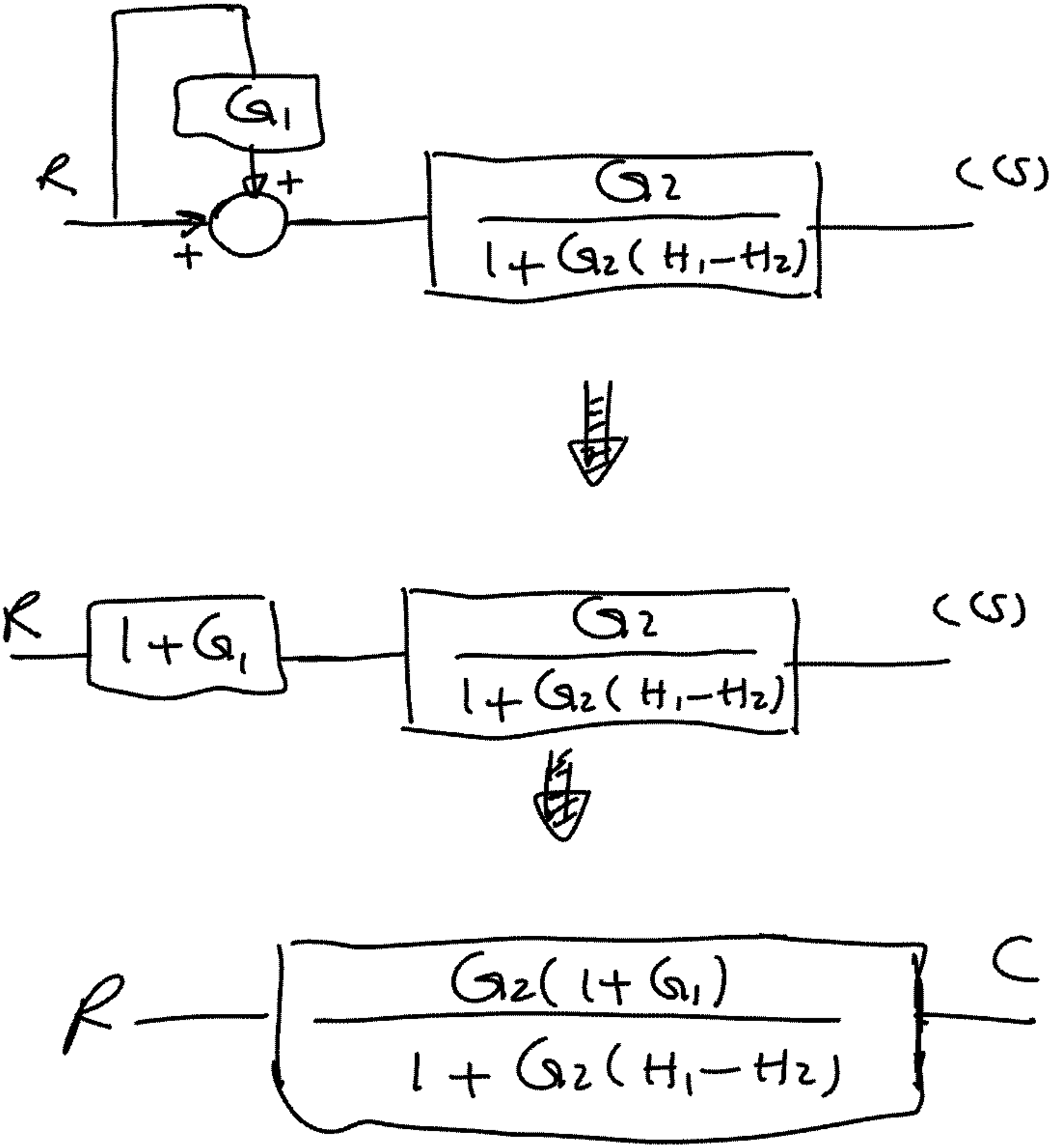




$\Downarrow$

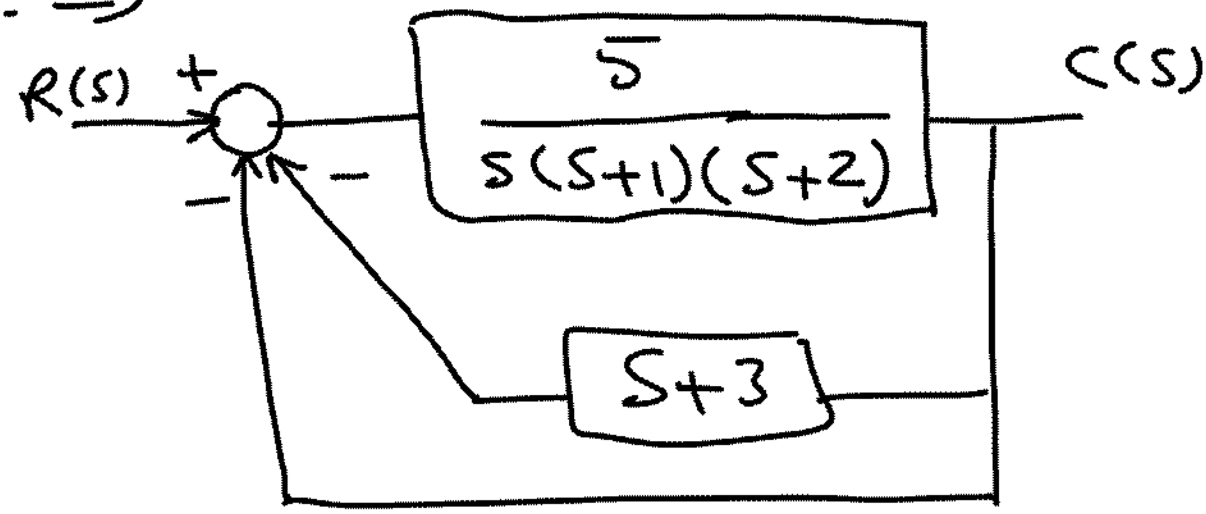


$\Downarrow$



$$\frac{C}{R} = \frac{G_1G_2 + G_2}{1 + G_2H_1 - G_2H_2}$$

Q3: $\Rightarrow$



$\Downarrow$

$$G = \frac{5}{s(s+1)(s+2) + 5(s+3)}$$

$$G = \frac{5}{s^3 + 3s^2 + 7s + 15}$$

- $K_p = \lim_{s \to 0} G(s) = \frac{5}{15} = 1/3$
- $K_v = \lim_{s \to 0} s\,G(s) = 0$
- $K_a = \lim_{s \to 0} s^2\,G(s) = 0$

(b) $SSE = \dfrac{50}{1+K_p} = \dfrac{3 \times 50}{4} = 37.5$

(c) $C/CS = s^3 + 3s^2 + 7s + 20$

$$
\begin{array}{c|cc}
s^3 & 1 & 7 \\
s^2 & 3 & 20 \\
s^1 & 1/3 & \\
s^0 & 20 &
\end{array}
$$

$\Rightarrow$ stable forever

"no condition"

(d) Type $\Rightarrow$ Zero type

Q4: $\Rightarrow$

$$G(s) = \frac{2s+10}{s^2}$$

$$T = \frac{2s+10}{s^2+2s+10}$$

$$\frac{Y}{R} = \frac{2s+10}{s^2+2s+10}$$

$$r(t) = \delta(t) \quad \Rightarrow \quad R = 1$$

$$Y = \frac{2(s+1)+8}{(s+1)^2+9}$$

$$T = 2\,\frac{(s+1)}{(s+1)^2+9} + \frac{8}{3}\,\frac{3}{(s+1)^2+9}$$

$\Downarrow$

$$y = 2e^{-t}\cos 3t + \frac{8}{3}e^{-t}\sin 3t$$

Q5: $\Rightarrow$

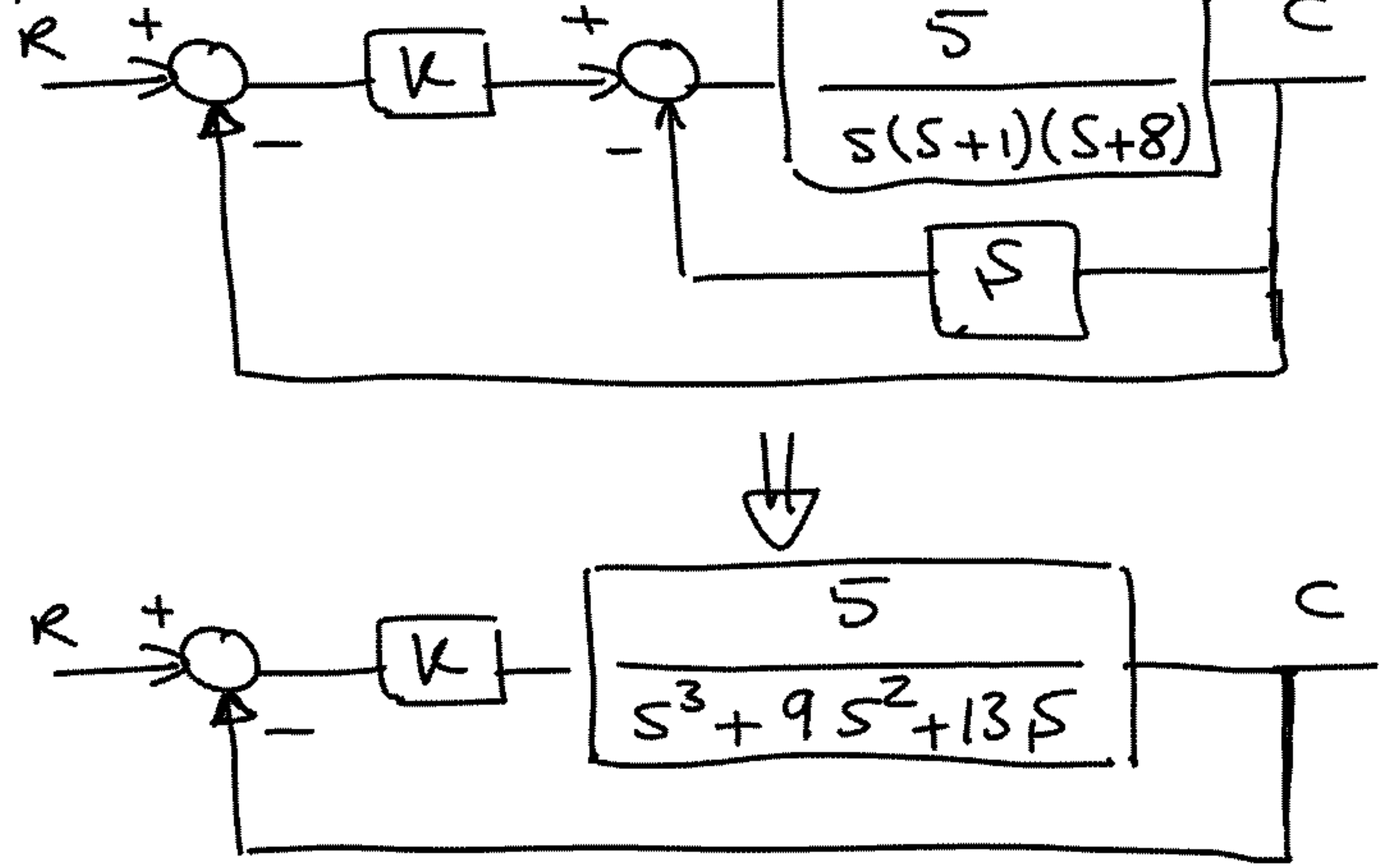
$$\frac{C(s)}{R(s)} = \frac{\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.05$$

$$t_s = \frac{4}{\xi\omega_n} \Rightarrow 2\% \text{ criteria}$$

$$\xi\omega_n = 2$$

Q6 ⇒



$$\frac{C}{R} = \frac{5K}{s^3 + 9s^2 + 13s + 5K}$$

$$\begin{array}{c|cc} s^3 & 1 & 13 \\ s^2 & 9 & 5K \\ s^1 & 13 - \frac{5K}{9} & \\ s^0 & 5K & \end{array}$$

$5K > 0 \Rightarrow \boxed{K > 0}$

$\Rightarrow 13 - \frac{5K}{9} > 0 \Rightarrow$

$\boxed{K < 23.4}$

$0 < K < 23.4$