Control engineering 1

model Ans

1st term 2015 / 2016

Q1.A

$$u(t) = \text{PI controller} * (h_d(t) - h_m(t))$$

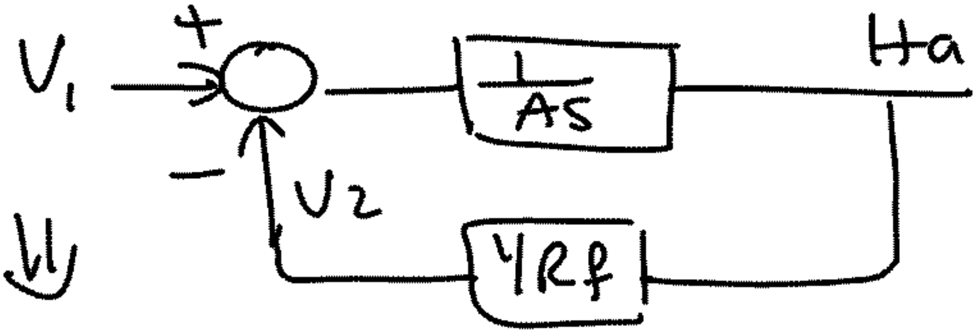
$$v_1(t) = K_v\, u(t)$$

$$\left.\begin{aligned} v_1(t) - v_2(t) &= A \frac{dh_a(t)}{dt} \\ v_2(t) &= \frac{h_a(t)}{R_f} \end{aligned}\right\}$$

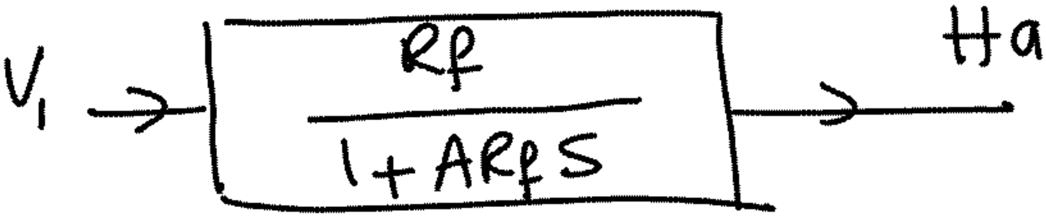
$$V_2 = \frac{H_a}{R_f}$$

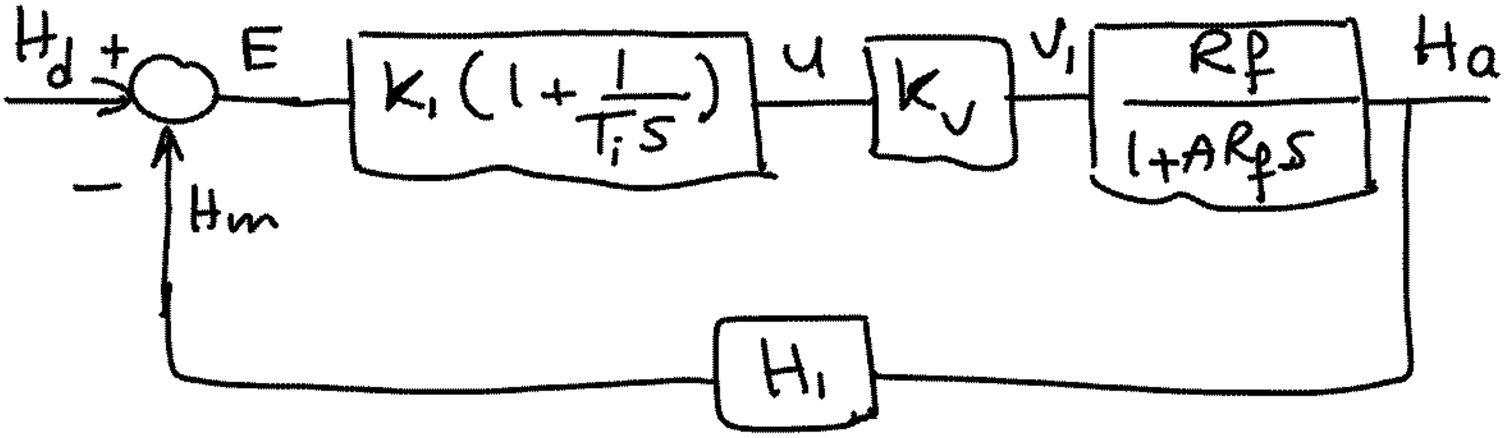
laplace

$$V_1 - V_2 = As\, H_a$$



$\Downarrow$





$$\frac{H_a}{H_d} = \frac{\dfrac{K_1 K_v R_f (1 + T_i s)}{(A R_f T_i s^2 + T_i s)}}{1 + \dfrac{K_1 K_v R_f H_1 (1 + T_i s)}{A R_f T_i s^2 + T_i s}}$$

$$\frac{H_a}{H_d} = \frac{K_1 K_v R_f \left((1 + T_i s)\right)}{(A R_f T_i) s^2 + T_i (1 + K_1 K_v R_f H_1) s + K_1 K_v R_f H_1}$$

Q1. B

putting $H_1 = 1$, then

$$\frac{H_a}{H_d} = \frac{1 + T_i s}{\left(\frac{A T_i}{K_1 K_v}\right) s^2 + T_i\left(\frac{1}{K_1 K_v R_f} + 1\right) s + 1}$$

$$\left(\frac{A T_i}{K_1 K_v}\right) = \frac{1}{\omega_n^2}$$

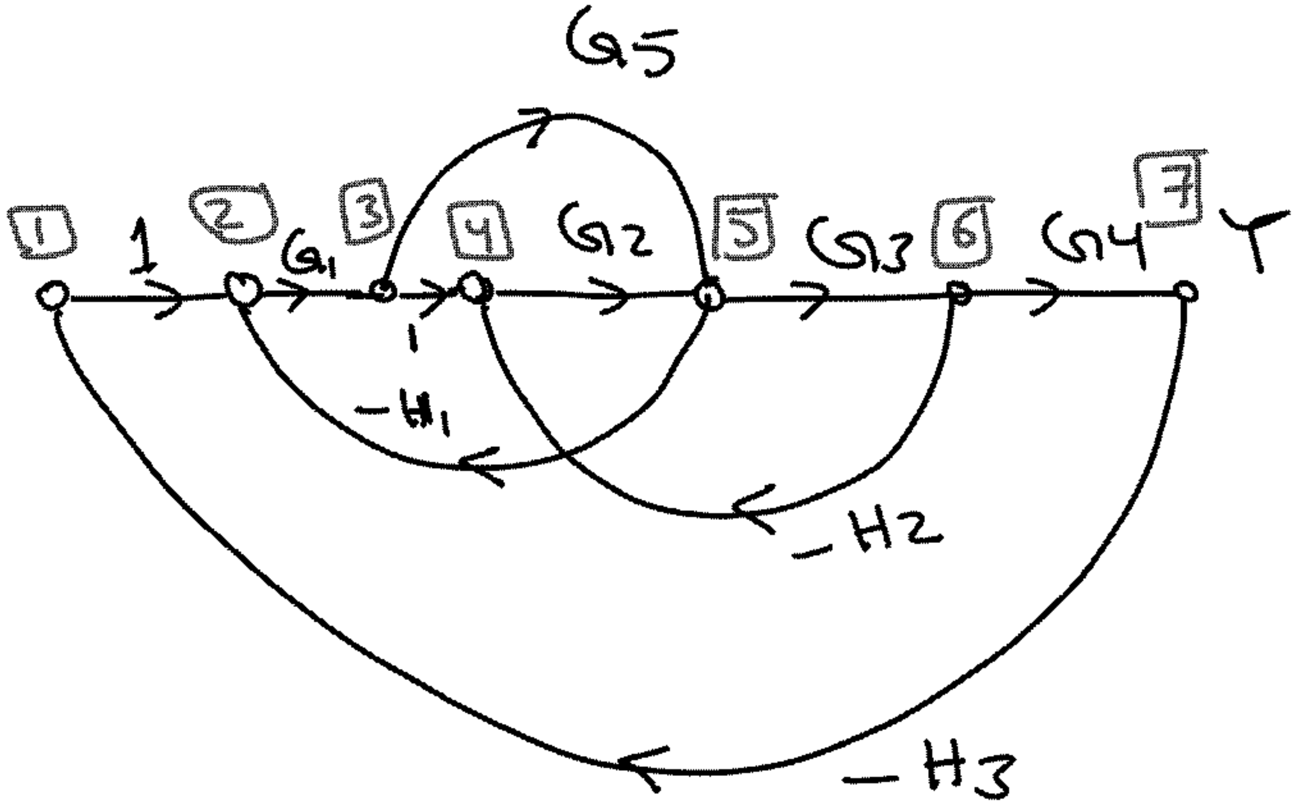
$$\therefore T_i = \frac{K_1 K_v}{\omega_n^2 A} = \frac{1 \times 0.1}{0.1^2 \times 2} = 5$$

$$\therefore T_i \left(\frac{1}{K_1 K_v R_f} + 1\right) = \frac{2\zeta}{\omega_n}$$

$$\zeta = \frac{\omega_n T_i}{2}\left(\frac{1}{K_1 K_v R_f} + 1\right)$$

$$\zeta = \frac{0.1 \times 5}{2}\left(\frac{1}{1 \times 0.1 \times 15} + 1\right) = 0.417$$

Q2.



path	gain
123567	$P_1 = G_1 G_5 G_3 G_4$
1234567	$P_2 = G_1 G_2 G_3 G_4$

loop	gain
1235671	$l_1 = -G_1 G_5 G_3 G_4 H_3$
12345671	$L_2 = -G_1 G_2 G_3 G_4 H_3$
2352	$l_3 = -G_1 G_5 H_1$
23452	$L_4 = -G_1 G_2 H_1$
4564	$l_5 = -G_2 G_3 H_2$

$$\Delta = 1 - (L_1 + l_2 + l_3 + L_4 + l_5) + (l_3 l_5)$$

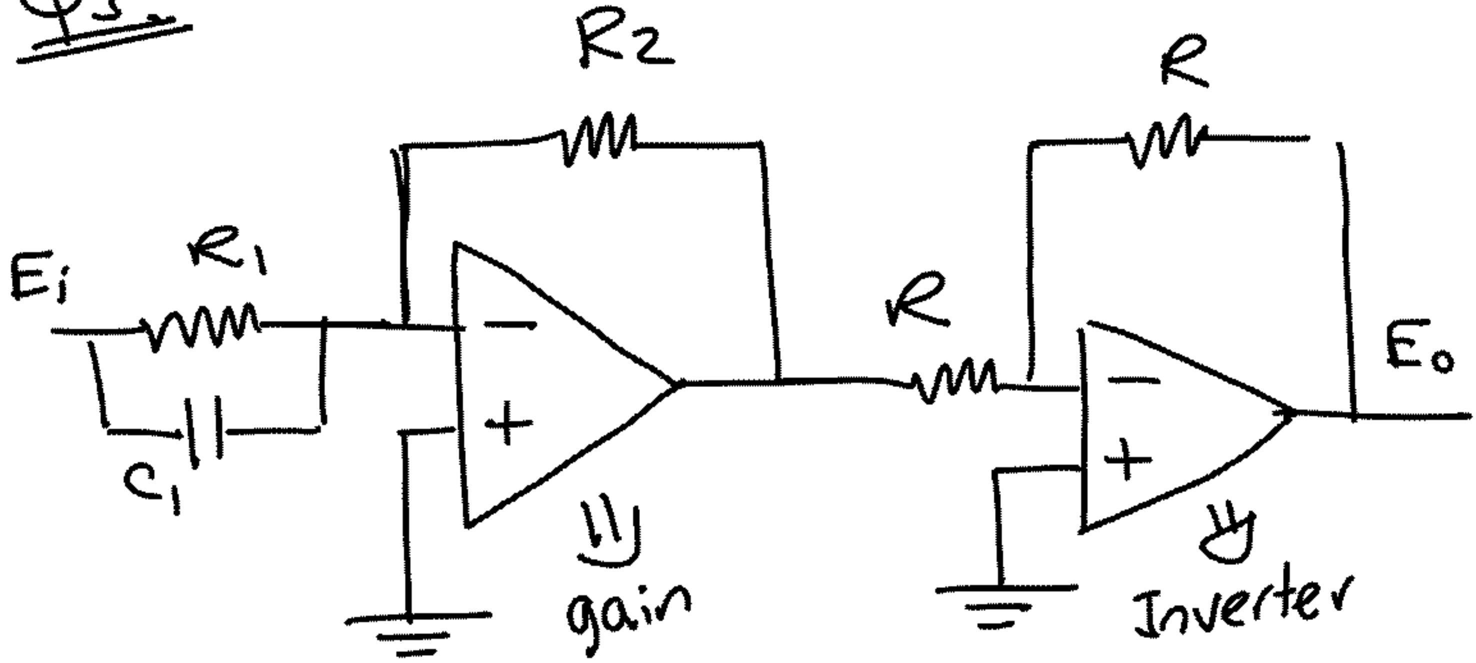
$$\Delta = 1 + G_1 G_5 G_3 G_4 H_3 + G_1 G_2 G_3 G_4 H_3 + G_1 G_5 H_1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_5 H_1 H_2$$

$$\Delta_1 = 1$$

$$\Delta 2 = 1$$

$$T.f = \frac{G_1 G_5 G_3 G_4 + G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_5 H_1 + G_1 G_2 G_3 G_4 H_3 + G_1 G_5 G_3 G_4 H_3 + G_1 G_2 G_3 G_5 H_1 H_2}$$

Q3.



$$\frac{E_o}{E_i} = \frac{Z_2}{Z_1}$$

$$Z_1 = \frac{R_1 / sC_1}{R_1 + 1/sC_1} = \frac{R_1}{1 + R_1 C_1 s}$$

$$Z_2 = R_2$$

$$\frac{E_o}{E_i} = \frac{R_2 (1 + R_1 C_1 s)}{R_1}$$

$$K_P = \frac{R_2}{R_1}$$

$$\frac{E_o}{E_i} = \frac{R_2}{R_1} + R_2 C_1 s \Rightarrow K_D = R_2 C_1$$

Q4

$$OS = \frac{1.25 - 1}{1} = 0.25$$

$$OS\% = 25\%$$

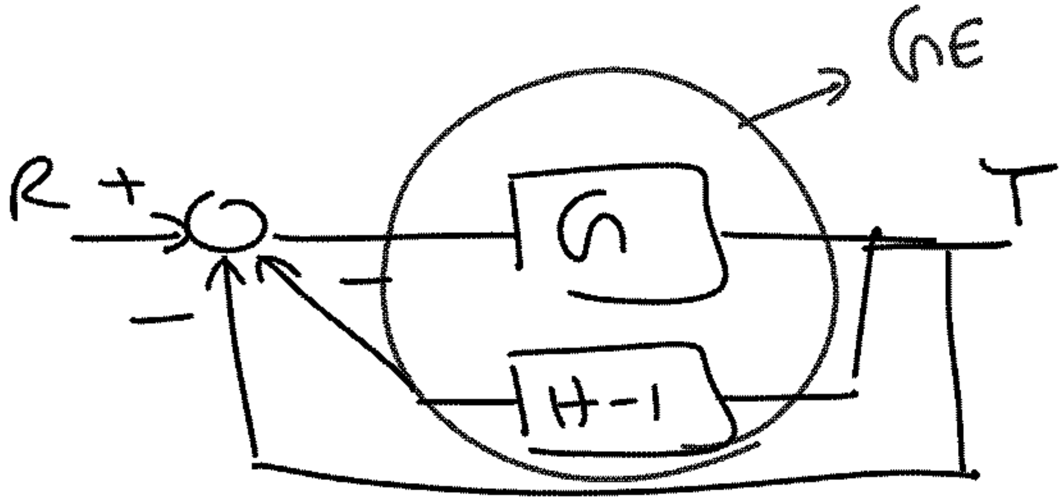
$$t_p = 0.01$$

$$\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}} = 0.404$$

$$\therefore \; t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$\therefore \; \omega_n = \frac{\pi}{t_p \sqrt{1 - \zeta^2}} = 343.4 \text{ rad/s}$$

Q5



$$\therefore T = \frac{GE}{1+GE}$$

$$\therefore T + T\,GE = GE$$

$$GE(1-T) = T$$

$$\therefore GE = \frac{T}{1-T}$$

T بسط

Assume $T = \dfrac{a}{b}$

T مقام

$$\therefore GE = \frac{\frac{a}{b}}{1-\frac{a}{b}} = \frac{\frac{a}{b}}{\frac{b-a}{b}}$$

$$GE = \frac{a}{b-a}$$

$$T = \frac{s+1}{(s+1)(s^2+s+2)+1}$$

$$G_E = \frac{s+1}{(s+1)(s^2+s+2)+1-(s+1)}$$

$$G_E = \frac{s+1}{s^3+2s^2+3s+2-s}$$

$$G_E = \frac{s+1}{s^3+2s^2+2s+2}$$

a

$K_p = G_E(0) = 1/2$

$K_v = 0$

$K_a = 0$

b

$$SSE = \frac{1}{1+1/2} = 2/3$$

c System must be stable

$c(s) = (s+1)(s^2+s+2)+1$

$$c/cs = s^3 + 2s^2 + 3s + 3$$

s^3	1	3
s^2	2	3
s^1	1.5	
s^0	3	

$\Rightarrow$ stable

(d) system type $\Rightarrow$ zero

مش محتاج GnE

Q6

$C/C(s) = 1 + G(s)H(s)$

$$= 1 + \frac{K(s+5)}{s(s+2)(1+Ts)}$$

$$C/C(s) = Ts^3 + (2T+1)s^2 + (K+2)s + 5K$$

$$\begin{array}{c|cc}
s^3 & T & K+2 \\
s^2 & 2T+1 & 5K \\
s^1 & \frac{(2T+1)(K+2) - 5KT}{2T+1} & \\
s^0 & 5K &
\end{array}$$

Condition for stability

$T > 0$

$T > -1/2$

$K > 0$

c

$K(1-3T) + 4T + 2 > 0$

$\therefore$ the condition for stability are

$T > 0$, $K > 0$, $K < \frac{4T+2}{3T-1}$