System model

Electric circuit

- electric circuits are frequently used in control systems largely because of the ease of manipulation and processing of electric signals.
- Although controllers are increasingly implemented with digital logic, many functions are still performed with analog circuit

① analog circuits are faster than digital, and for very simple controllers, an analog circuit would be less expensive than a digital implementation

② furthermore, the power amplifier for electromechanical control and the anti-alias prefilters for digital control must be analog circuit

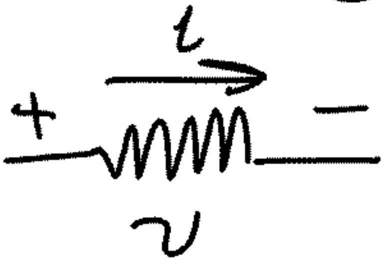
Kirchhoff's laws

① Kirchhoff's Current law KCL
the algebraic sum of currents leaving a junction or node equals the algebraic sum of currents entering that node

② Kirchhoff's voltage law KVL
the algebraic sum of all voltages taken around a closed path in a circuit is zero

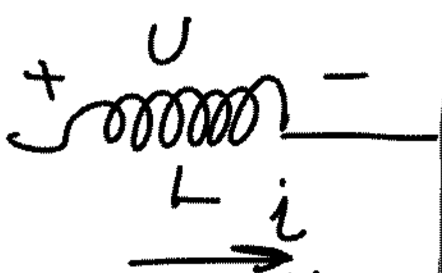
<u>i.e.</u> ① $I_{in} = I_{out}$

② $\sum$ voltage sources $= \sum$ voltage drops



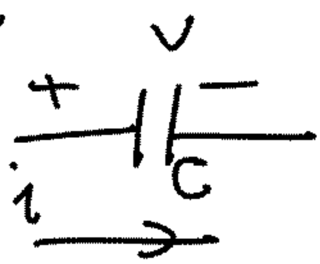
$v(t) = i(t) * R$

$V(\omega) = I(\omega) R$



$v = L \frac{di}{dt}$

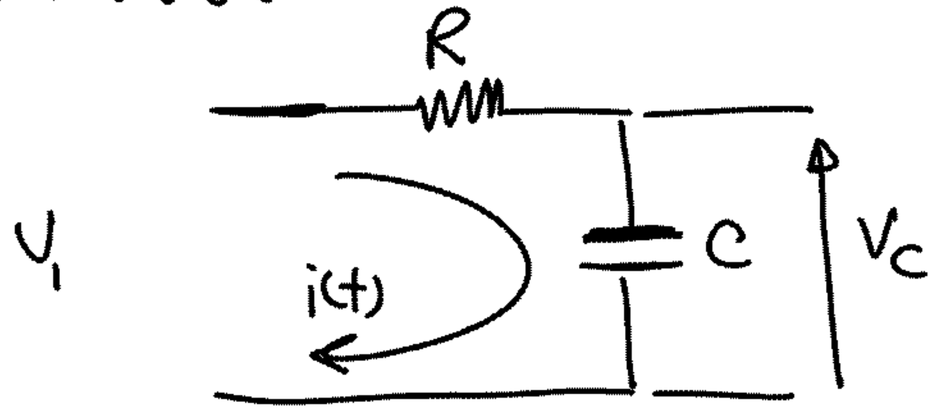
$V = SLI$



$v = \frac{1}{C} \int i \, dt$

$V = I/CS$

RLC Circuit



$$v_1(t) = i(t)R + \frac{1}{C}\int i(t)\,dt$$

$\downarrow$ taking laplace

$$V_1(s) = I(s)R + \frac{I(s)}{Cs}$$

$$\Downarrow$$

$$V_1 = IR + \frac{I}{Cs} = I\left(R + \frac{1}{Cs}\right) \rightarrow 1$$

$$v_c(t) = \frac{1}{C}\int i(t)\,dt$$

$$\Downarrow$$

$$V_c = \frac{I}{Cs} \rightarrow 2$$

divide 2 by 1

$$\therefore \quad \frac{V_c}{V_1} = \frac{1/CS}{R + 1/CS}$$

$$\frac{V_c}{V_1} = \frac{1}{1 + RCS}$$

mathematically $\Rightarrow$ analytically

$$\therefore \quad \frac{V_c}{V_1} = \frac{1/RC}{S + 1/RC}$$

Assume $v_1(t) = u(t)$  1 volt

$$V_1(t) = u(t) \Longrightarrow V_1 = 1/S$$

$$V_c = \frac{1}{RC} * \frac{1}{S(S + 1/RC)}$$

$$\therefore \quad V_c = \frac{1}{RC}\left(\frac{RC}{S} - \frac{RC}{S + 1/RC}\right)$$

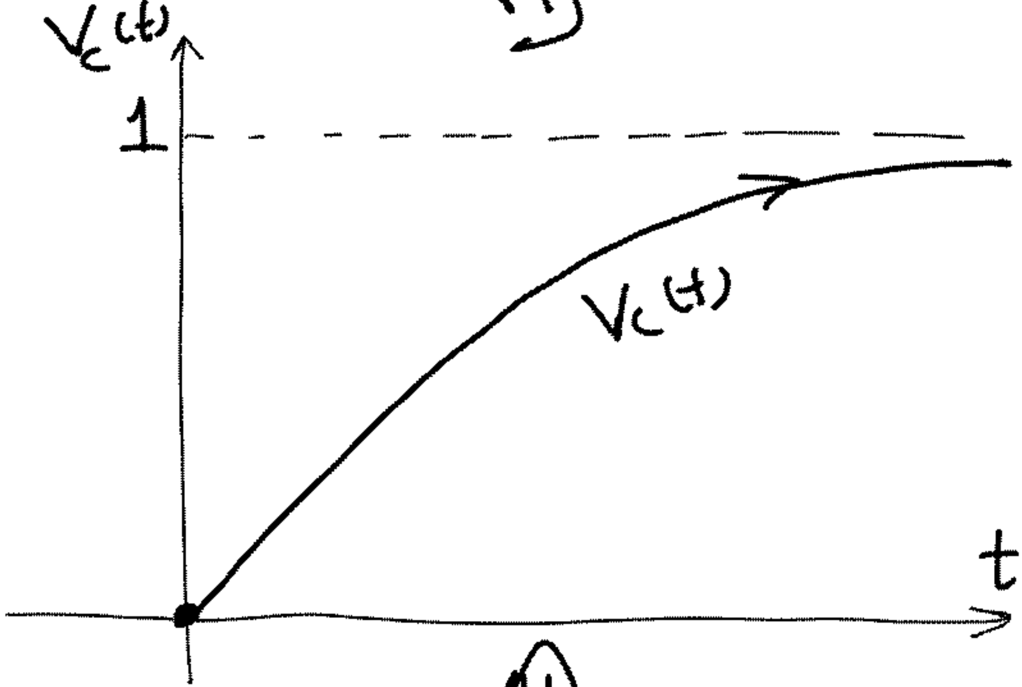
$$V_c = \frac{1}{s} - \frac{1}{s + 1/RC}$$

$\Downarrow$

$$V_c(t) = 1 - e^{-\frac{1}{RC}t}$$

Assume $R = 1$, $C = 0.0001$

$\therefore$ $V_c(t) = 1 - e^{-10000t}$ like this



$\Uparrow$

charging capacitor

② by MATLAB ⇒ .m file

$$\frac{V_c}{V_1} = \frac{1}{1 + RCs}$$

Assume

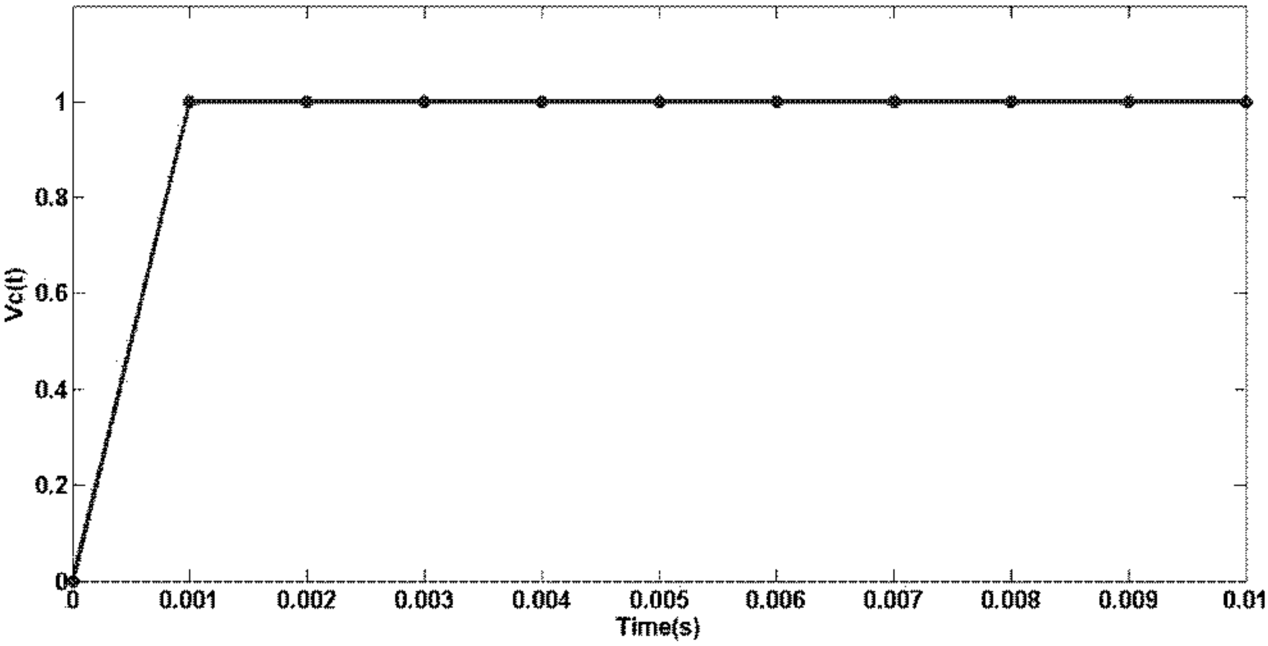
⇓

$V_1 = u(t) \Rightarrow$ unit step

$R = 1\ \Omega$

$C = 0.0001$

```
R = 1;
C = 0.0001;
num = 1;
den = [R*C ▽ 1];
sys = tf(num, den);
t = 0:0.001:0.01;
Vc = step(sys, t);
plot(t, Vc)
```



③ MATLAB // SIMULink

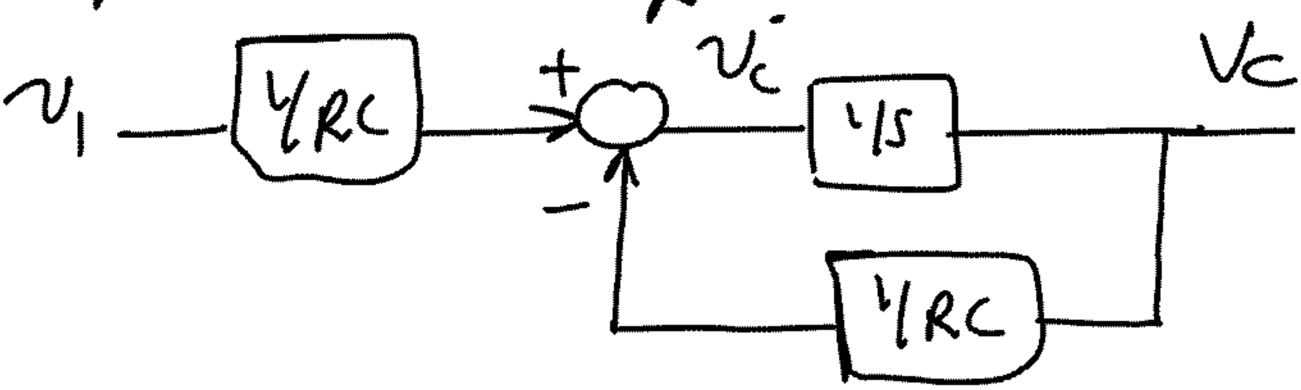
$$\frac{V_c}{V_1} = \frac{1}{1+RCs}$$

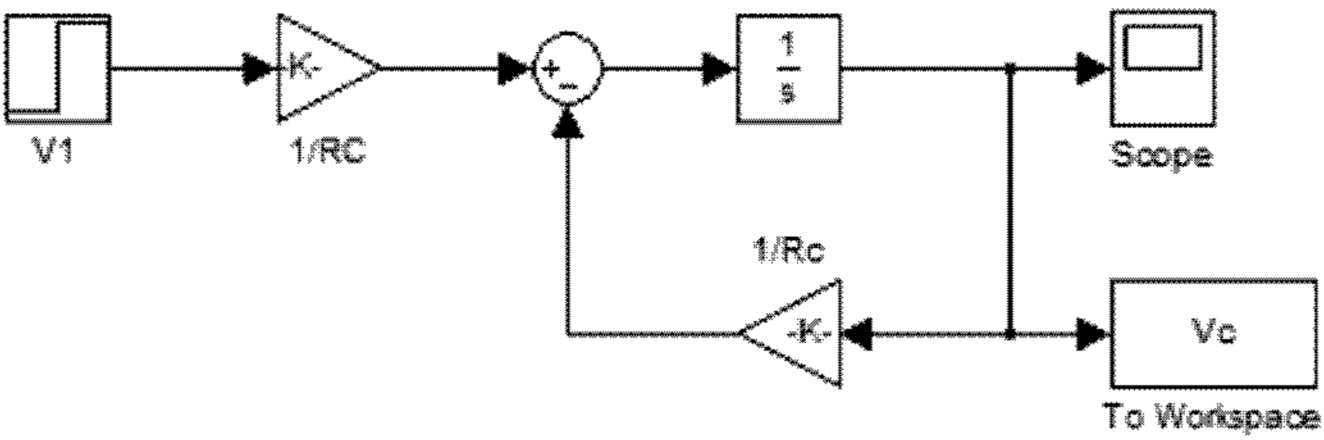
$\Downarrow$ taking inverse laplace

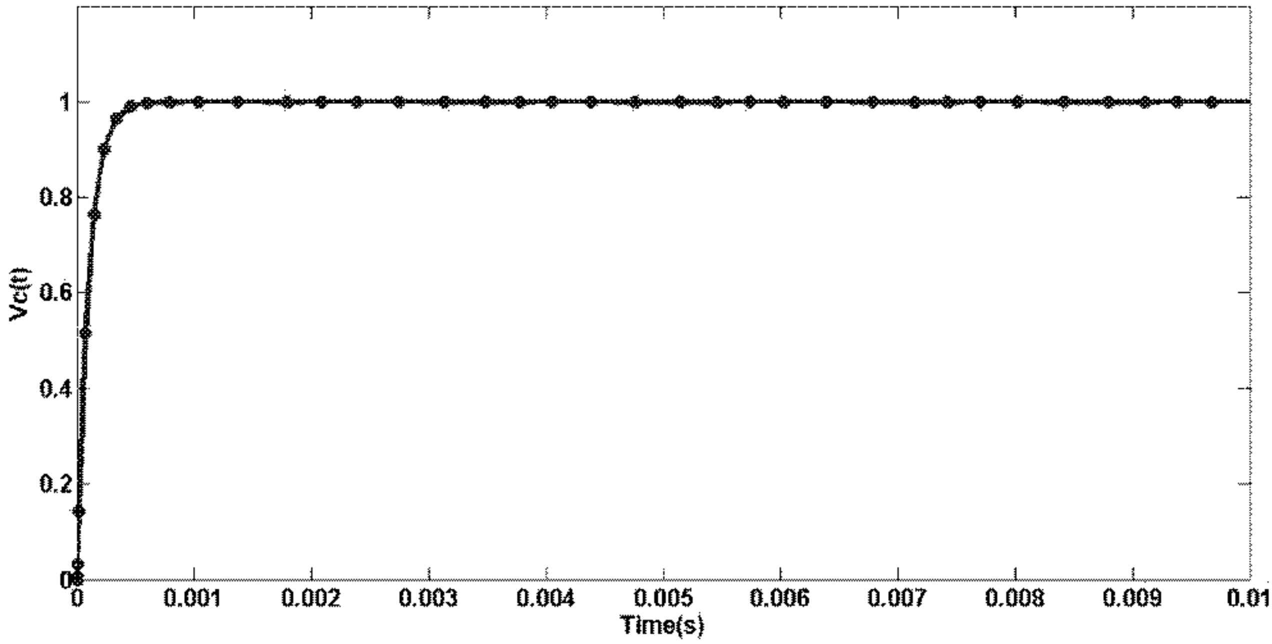
$$v_c + RC\,\dot{v}_c = v_1$$

$\downarrow$

$$\frac{1}{RC}v_c + \dot{v}_c = \frac{1}{RC}v_1$$

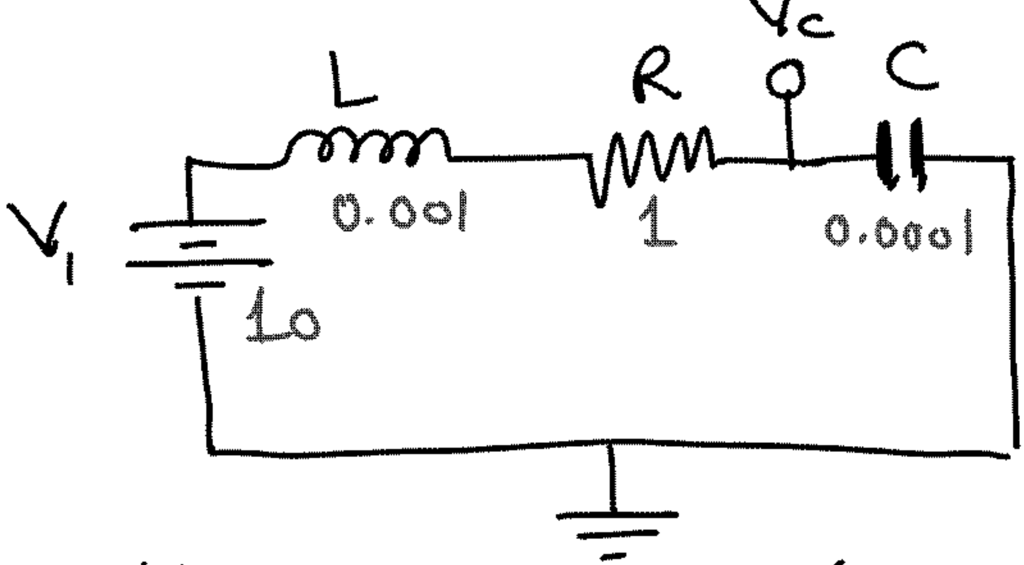






Ex ⇒ RLC Circuit

use the SIMulink to solve the following RLC Circuit



$$v_1(t) = L\frac{di(t)}{dt} + i(t) * R + \frac{1}{C}\int i(t)\,dt$$

⇓ take laplace

$$V_1 = LSI + IR + \frac{I}{CS}$$

$$V_1 = I\left(R + LS + \frac{1}{CS}\right) = \frac{LCS^2 + RCS + 1}{CS}$$

1

$$v_c(t) = \frac{1}{C}\int i(t)\,dt$$

$$\Downarrow$$

$$V_c = \frac{I}{CS} \longrightarrow 2$$

Divide 2 by 1

$$\therefore \quad \frac{V_c}{V_1} = \frac{1/CS}{(LCS^2 + RCS + 1)/CS}$$

$$\therefore \quad \frac{V_c}{V_1} = \frac{1}{LCS^2 + RCS + 1}$$

$$\frac{V_c}{V_1} = \frac{1/LC}{S^2 + \frac{R}{L}S + \frac{1}{LC}}$$

Assume $v_1 = 10\,u(t)$, $R=1$, $C=0.0001$, $L=0.001$

$$\frac{V_c}{V_1} = \frac{10{,}000{,}000}{S^2 + 1000\,S + 10000000}$$

$\Downarrow$ (1) Analytically

$$V_C = \frac{10^8}{S(S^2 + 1000\,S + 10^7)}$$

$\Downarrow$ partial fraction

$$\frac{10^8}{S(S^2 + 1000\,S + 10^7)} = \frac{10}{S} + \frac{AS + B}{S^2 + 1000\,S + 10^7}$$

$\Downarrow$

$$10^8 = 10\left(S^2 + 1000\,S + 10^7\right) + S(AS + B)$$

Coefficient of $S = 0$

$$0 = 1000 + B \Rightarrow B = -1000$$

Coefficient of $S^2 = 0 = 10 + A \Rightarrow A = -10$

$$\therefore V_c = \frac{10}{s} - \frac{10s + 1000}{s^2 + 1000s + 10^7}$$

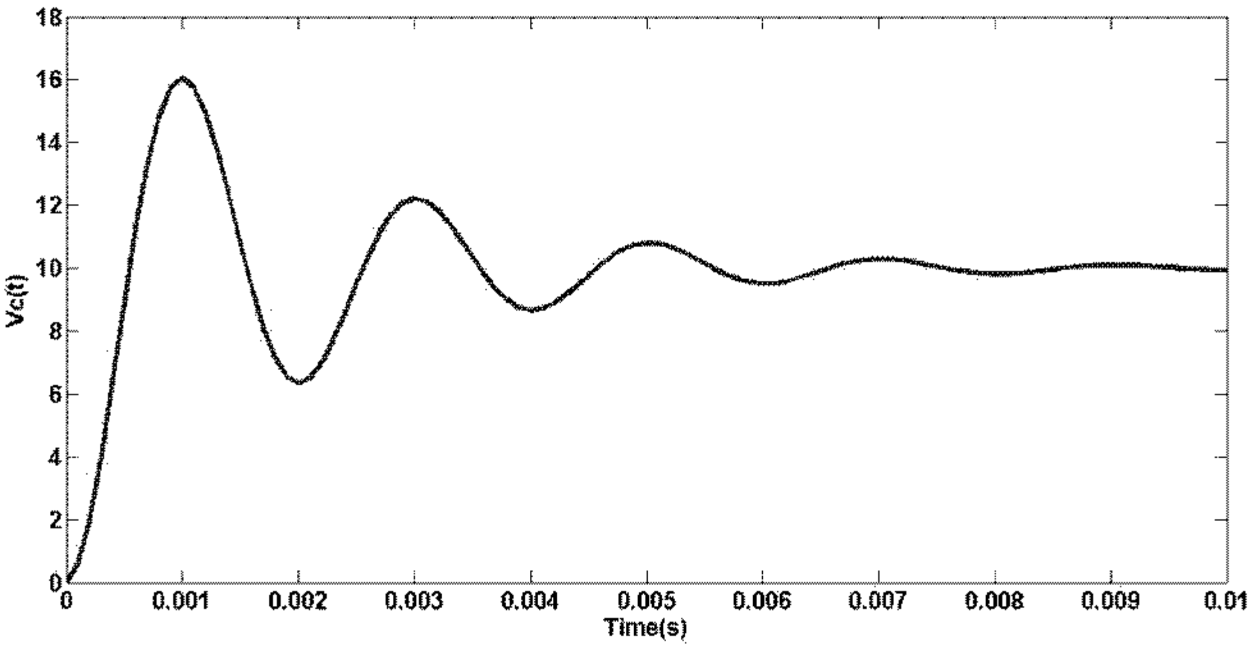
$$\Downarrow$$

$$V_c = \frac{10}{s} - \frac{10(s+500) - 4000}{(s+500)^2 + 9750000}$$

$$V_c = \frac{10}{s} - 10\frac{s+500}{(s+500)^2 + 9750000} + \frac{4000}{(s+500)^2 + 9750000}$$

$\Downarrow$ inverse laplace

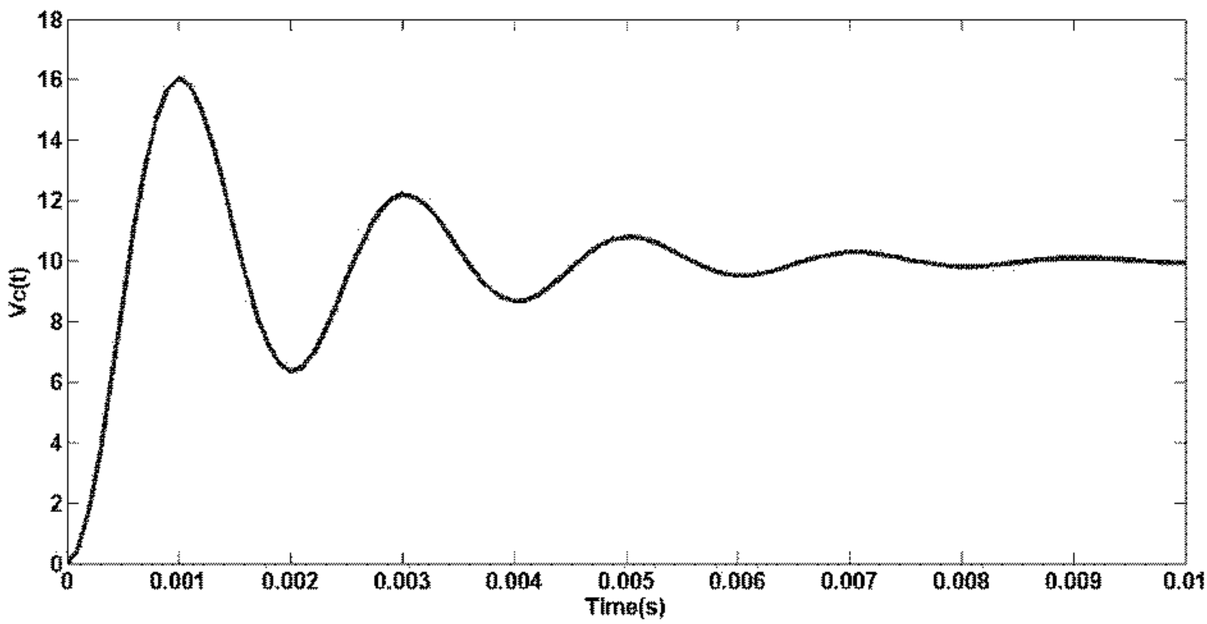
$$v_c(t) = 10 - 10e^{-500t}\cos\sqrt{9750000}\,t - \frac{4000}{\sqrt{}}e^{-500t}\sin\sqrt{}$$



(2) By MATLAB // .m file

$$\frac{V_c}{V_i} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

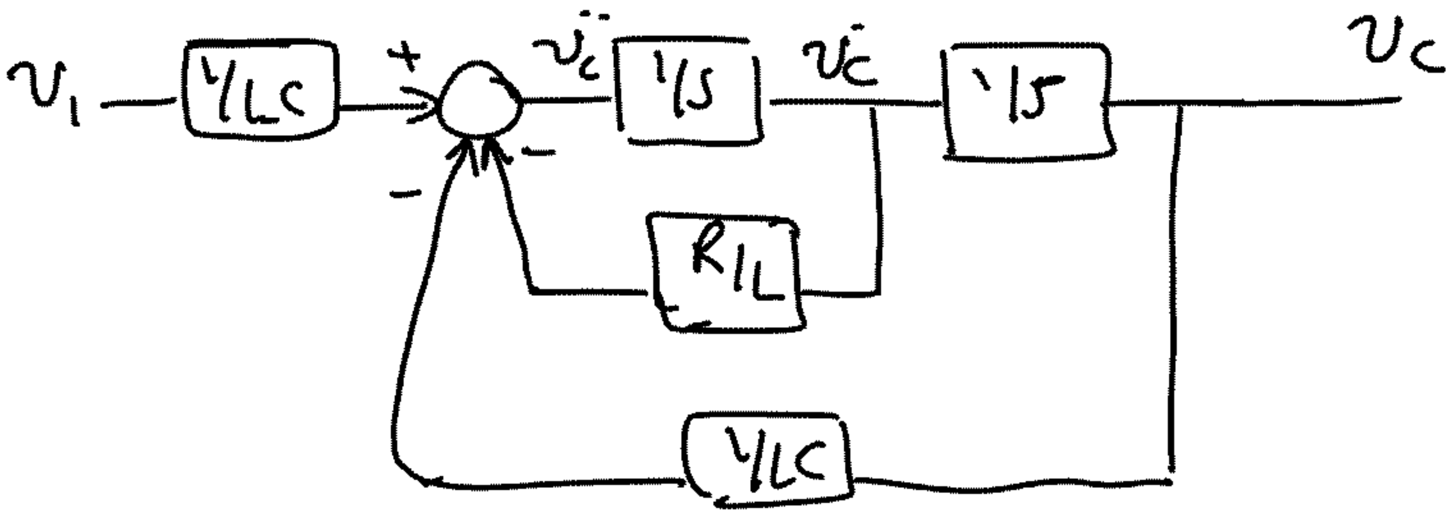
```
R = 1;
L = 0.001;
C = 0.0001;
num = 1/(L*C);
den = [1  R/L  1/(L*C)];
sys = tf(num, den);
t = 0:0001:0.01;
Vc = step(10*sys, t)
plot(t, Vc)
```

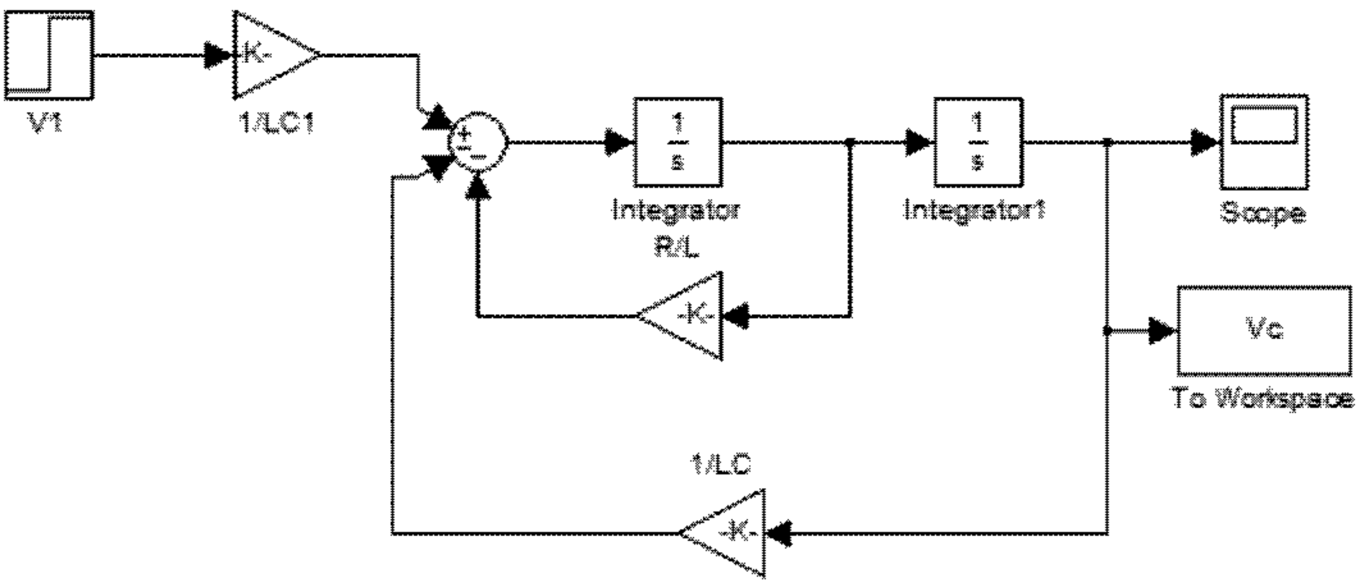


③ By SIMulink

$$\frac{V_c}{V_1} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\ddot{v}_c + \frac{R}{L}\dot{v}_c + \frac{1}{LC}v_c = \frac{1}{LC}v_1$$





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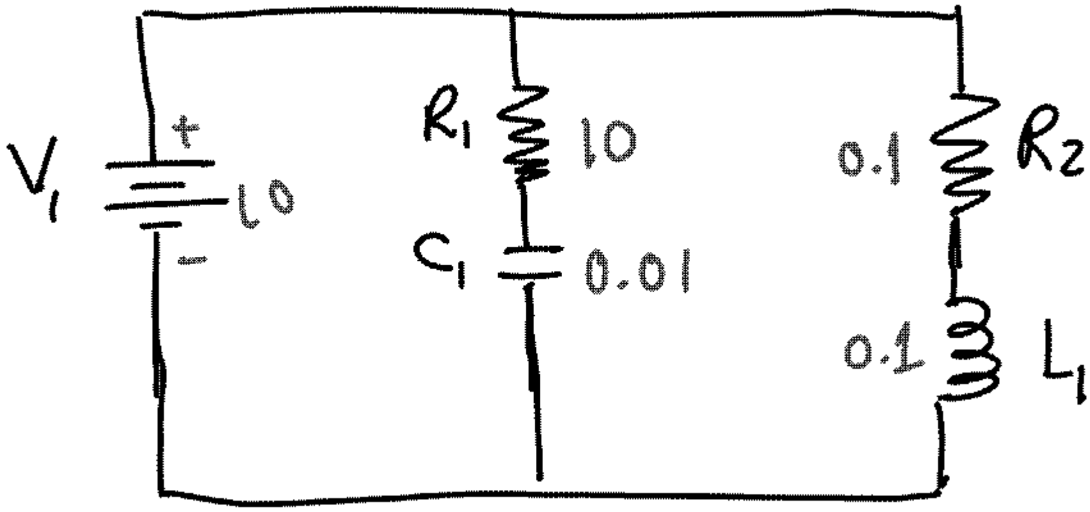
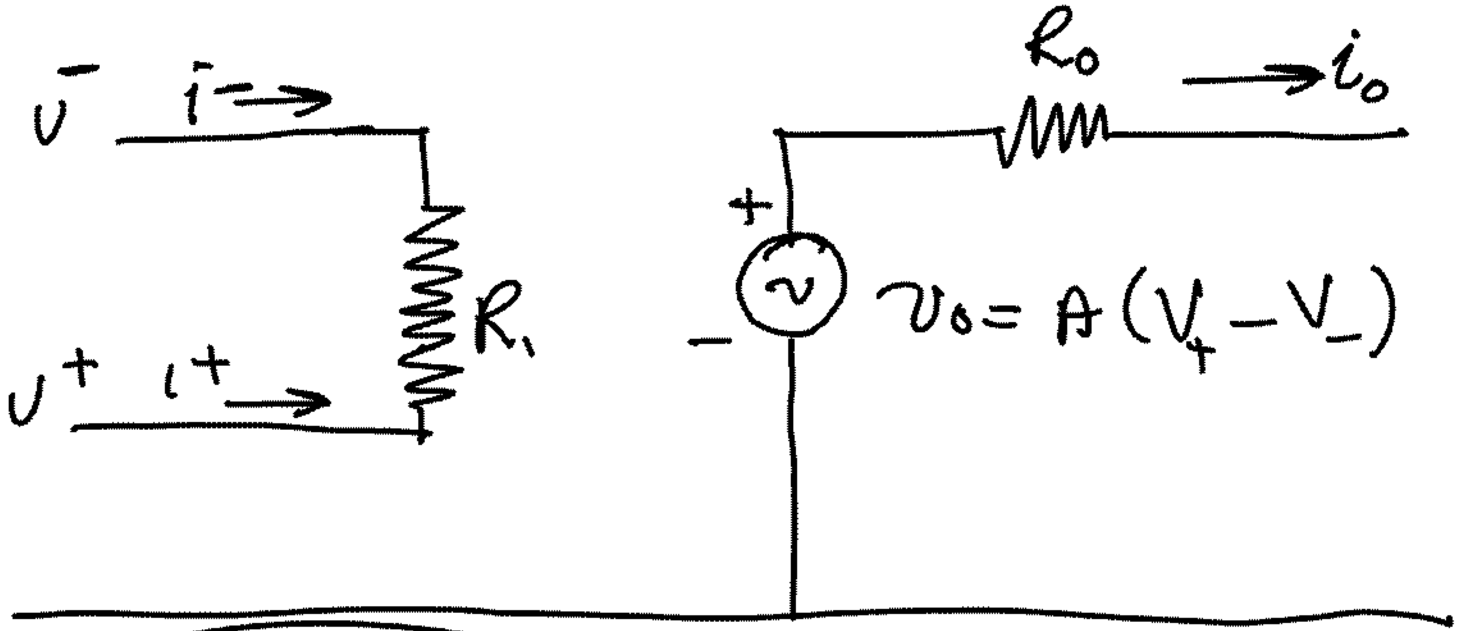
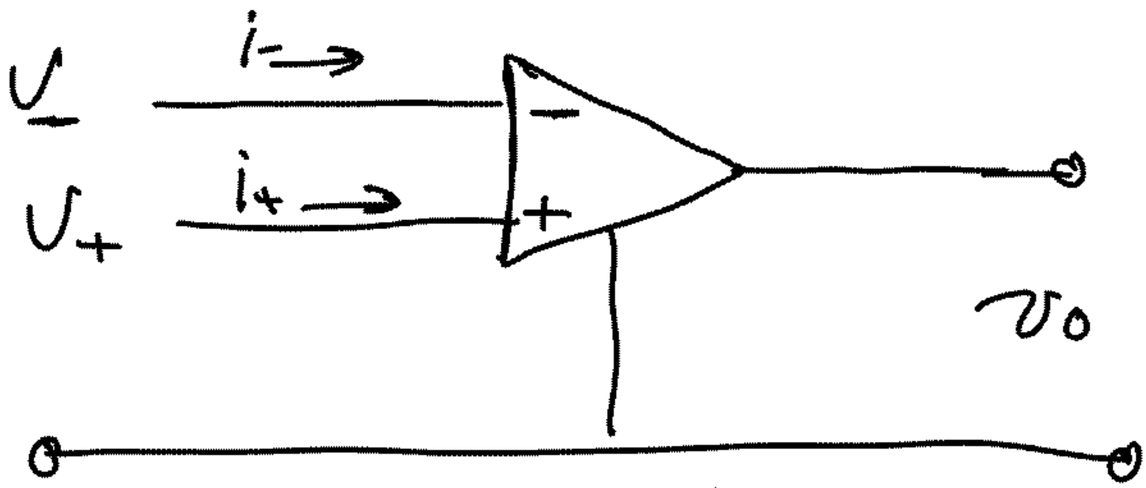


operational amplifier ⇒

the simplified circuit of the op-amp is shown

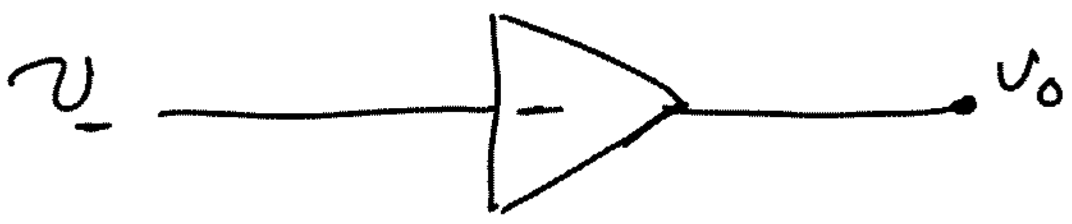


schematic symbol



if the positive terminal is not shown, it is assumed to be connected to ground, $v_+ = 0$

and the reduced symbol below is used,



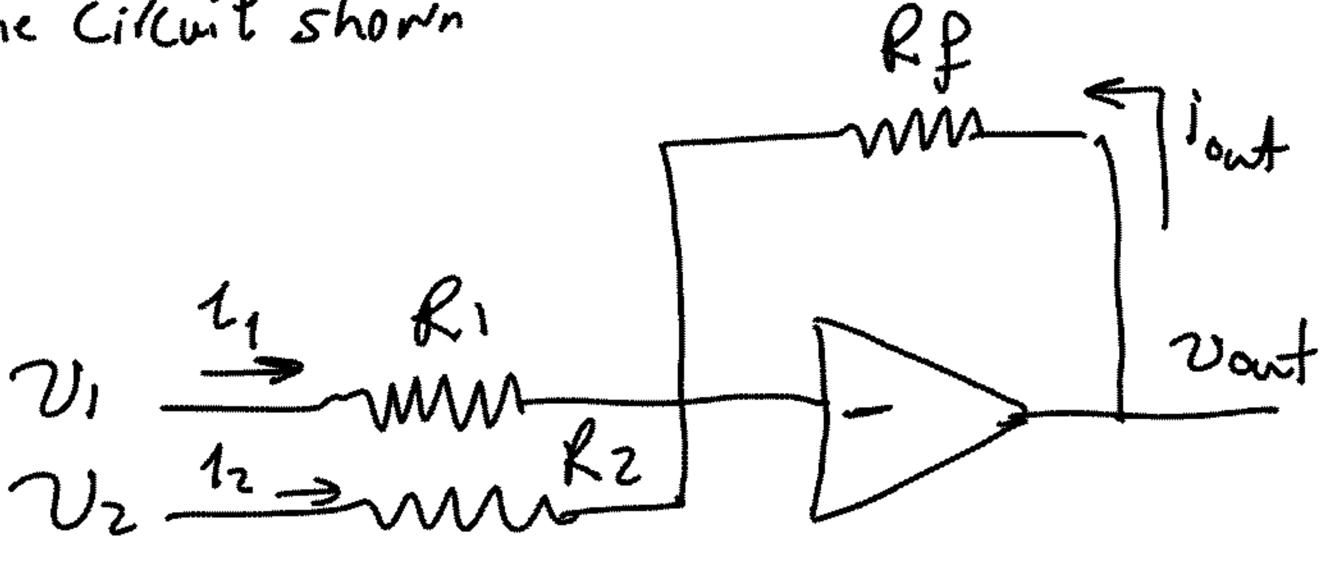
it is usually assumed that the op_amp is ideal with the values $R_i = \infty$, $R_o = 0$, $A = \infty$

$\therefore$ ① $i_+ = i_- = 0$

② $v_+ = v_-$

Ex

find the equations and transfer function of the circuit shown



$\therefore\ v_+ = 0 \Rightarrow v_- = 0$

$$i_1 = \frac{v_1}{R_1}, \quad i_2 = \frac{v_2}{R_2}, \quad i_{out} = \frac{v_{out}}{R_f}$$

$$i_1 + i_2 + i_3 = 0$$

$$\Downarrow$$

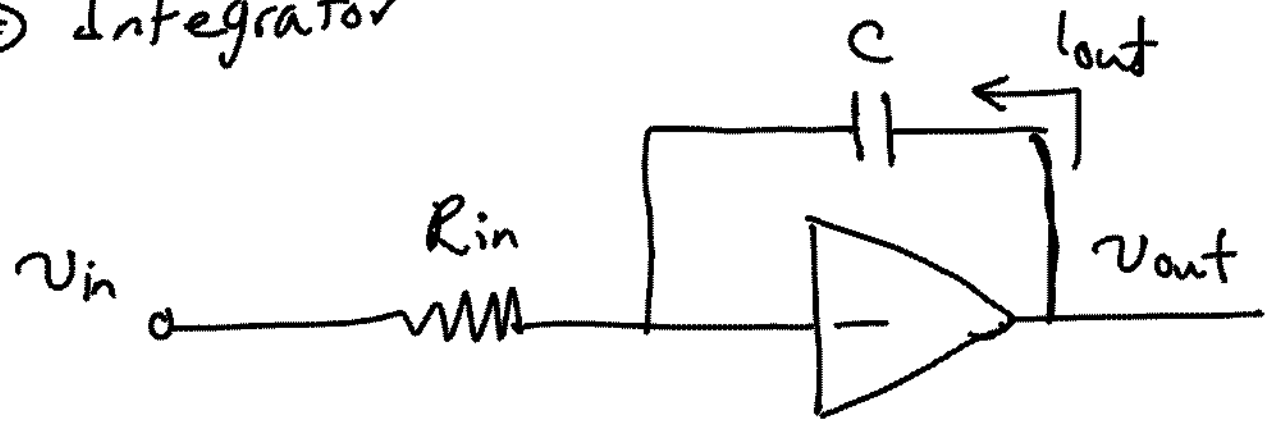
$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_{out}}{R_f} = 0$$

$$\therefore\ v_{out} = -\left[\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2\right]$$

the circuit o/p is a weighted sum of the input voltages with a sign change.

$\Downarrow$ Summer circuit

Ex $\Rightarrow$ Integrator



$\therefore\ v_+ = 0 \Rightarrow v_- = 0$

$$\frac{v_{in}}{R_{in}} + C\frac{dv_{out}}{dt} = 0$$

$$\therefore\ \frac{dv_{out}}{dt} = -\frac{v_{in}}{CR_{in}}$$

$$\therefore\ v_{out} = \frac{-1}{R_{in}C}\int_0^t v_{in}(\tau)\,d\tau + v_{out}(0)$$

OR

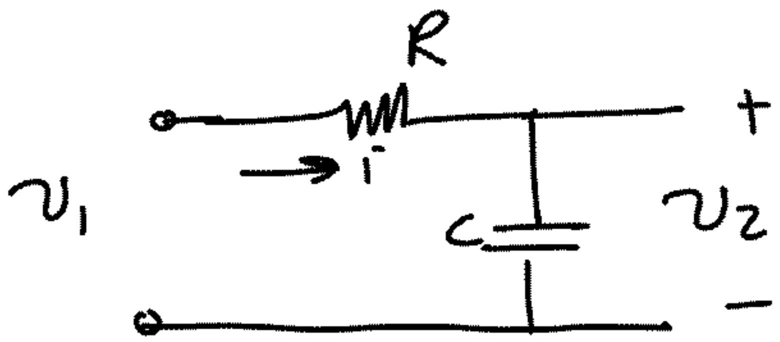
$$sV_{out} = \frac{-1}{R_{in}C}v_{in}$$

$$\therefore\ \frac{V_{out}}{v_{in}} = \frac{-1}{sR_{in}C}$$

Prob.

Do it again assuming $u(t) = t$

Ex $\Rightarrow$



$$v_1(t) = i(t) \cancel{\times} R + \frac{1}{C}\int i(t)\,dt$$

$$v_2(t) = \frac{1}{C}\int i(t)\,dt$$

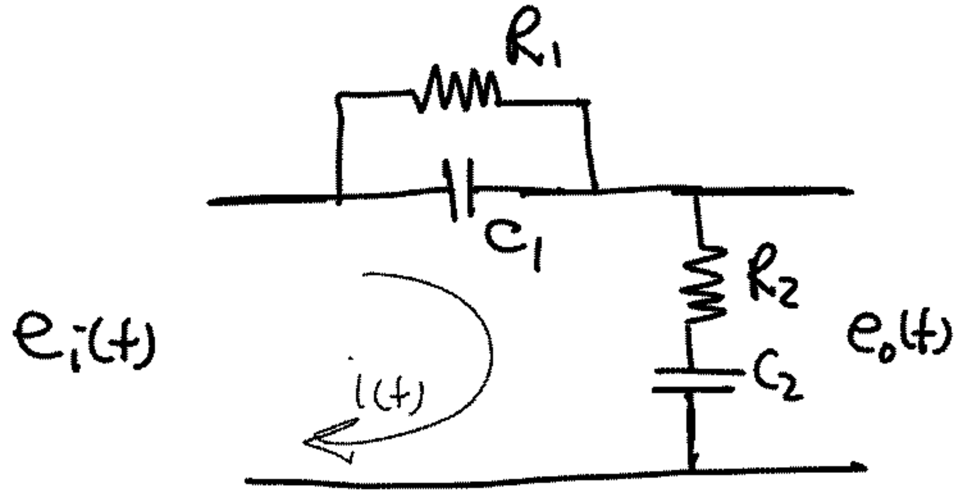
$\Downarrow$

$$V_1 = IR + \frac{I}{Cs} = I\left(R + \frac{1}{Cs}\right)$$

$$V_2 = I/Cs$$

$$\frac{V_2}{V_1} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1 + RC_1 s}$$

Ex $\Rightarrow$



$$Z_1 = \frac{R_1 / sC_1}{R_1 + 1/sC_1}$$

$$= \frac{R_1}{1 + sC_1 R_1}$$

$$Z_2 = R_2 + 1/sC_2 = \frac{1 + sC_2 R_2}{sC_2}$$

$$E_i = I(s)\left[\frac{R_1}{1 + sC_1 R_1} + \frac{1 + sC_2 R_2}{sC_2}\right]$$

$$E_i = I(s)\left[\frac{sC_2R_1 + (1+sC_2R_2)(1+sC_1R_1)}{sC_2(1+sC_1R_1)}\right]$$

$$E_o = I(\omega)\left(\frac{1+sC_2R_2}{sC_2}\right)$$

$$\therefore \frac{E_o}{E_i} = \frac{(1+sC_1R_1)(1+sC_2R_2)}{sC_2R_1 + (1+sC_2R_2)(1+sC_1R_1)}$$

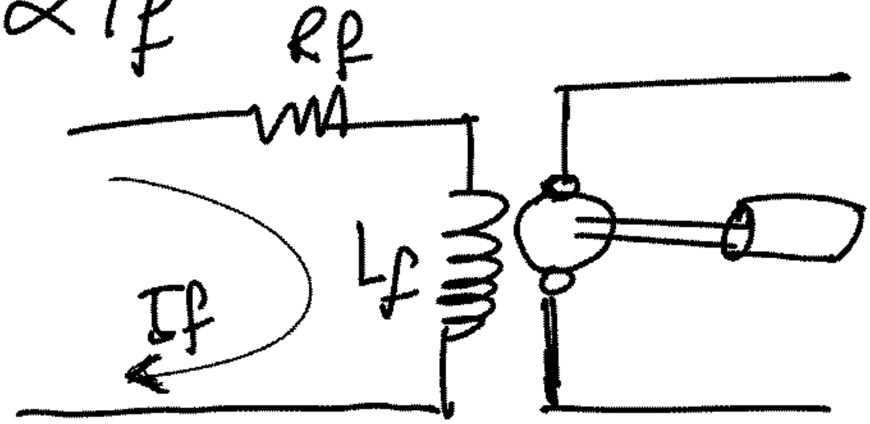
DC-Motor

Torque $\propto i_a i_f$

(1) Field Control

i_a: Constant

$T \propto i_f$



$$V_f = I_f(R_f + sL_f)$$

$$T = K I_f$$

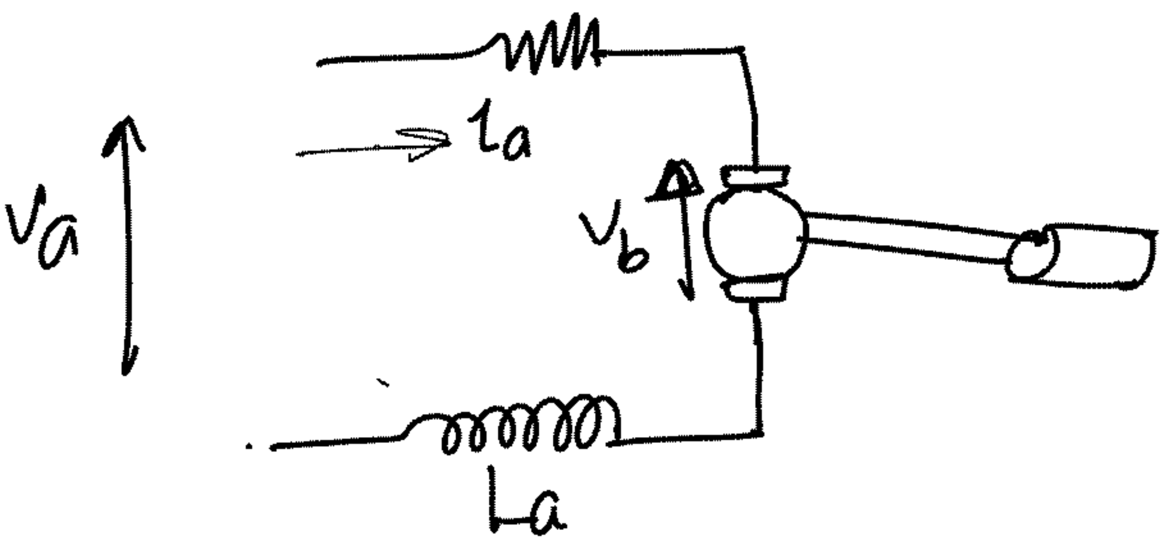
$$T = (Is^2 + bs)\theta$$

$$V_f \longrightarrow \boxed{\frac{1}{R_f + sL_f}} \xrightarrow{I_f} \boxed{K} \xrightarrow{T} \boxed{\frac{1}{Is^2 + bs}} \longrightarrow \theta$$

$$\theta / V_f = \frac{K}{(R_f + sL_f)(Is^2 + bs)}$$

② Armature Control i_f : constant

$T \propto i_a$



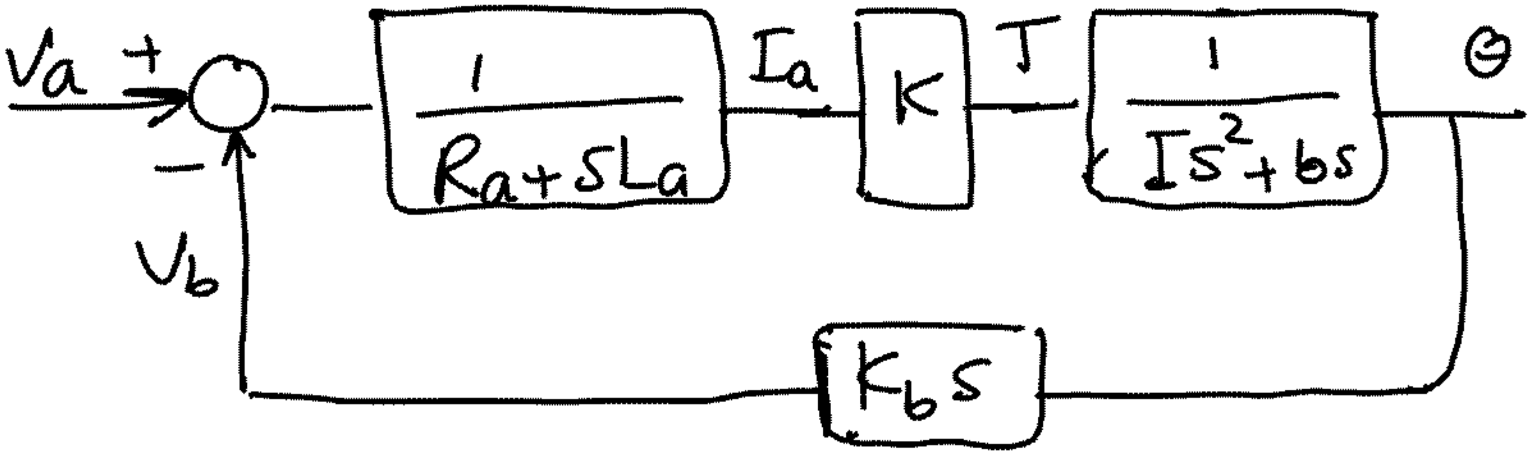
$$V_a - V_b = I_a(R + sL_a)$$

$$V_b \propto \dot{\theta}$$

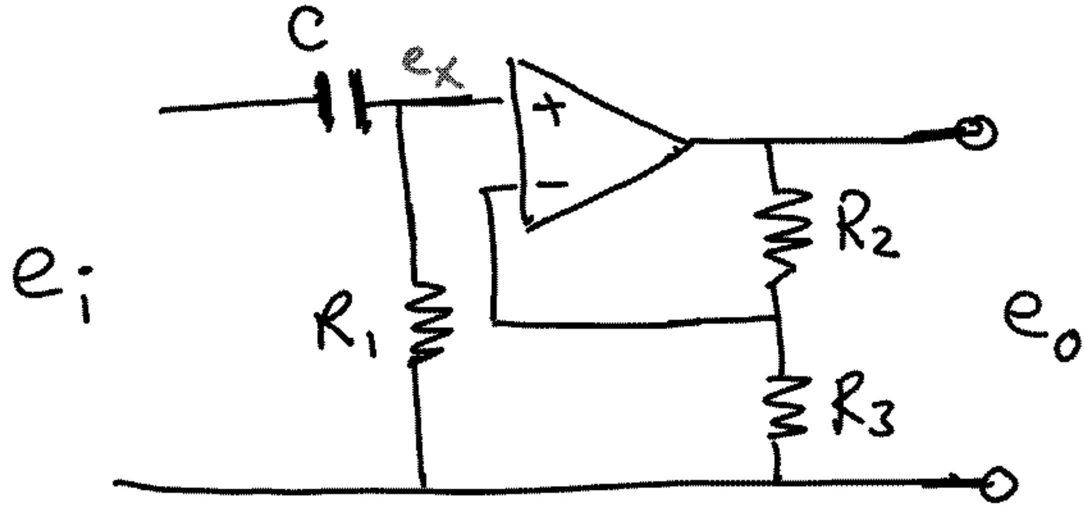
$$V_b = K_b s \theta$$

$$T = K I_a$$

$$T = (Is^2 + bs)\theta$$



Ex ⇒



Find E_o / E_i

Sol ⇒

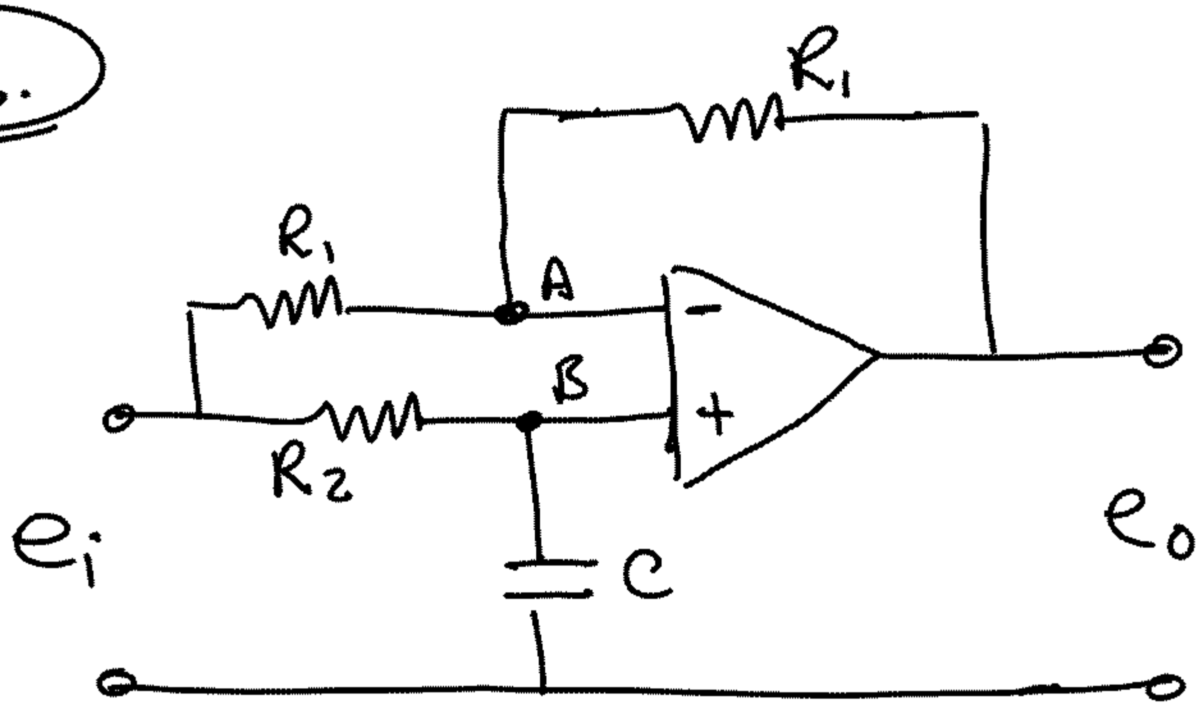
$$\frac{E_x}{E_i} = \frac{R_1}{R_1 + 1/SC}$$

$$\frac{E_x}{E_o} = \frac{R_3}{R_2 + R_3}$$

$$\frac{E_o}{E_i} = \frac{R_1}{1/SC + R_1} * \frac{R_3 + R_2}{R_3}$$

$$= \frac{R_1 (R_3 + R_2)}{R_3 (R_1 + 1/SC)}$$

Prob.



Find E_o / E_i