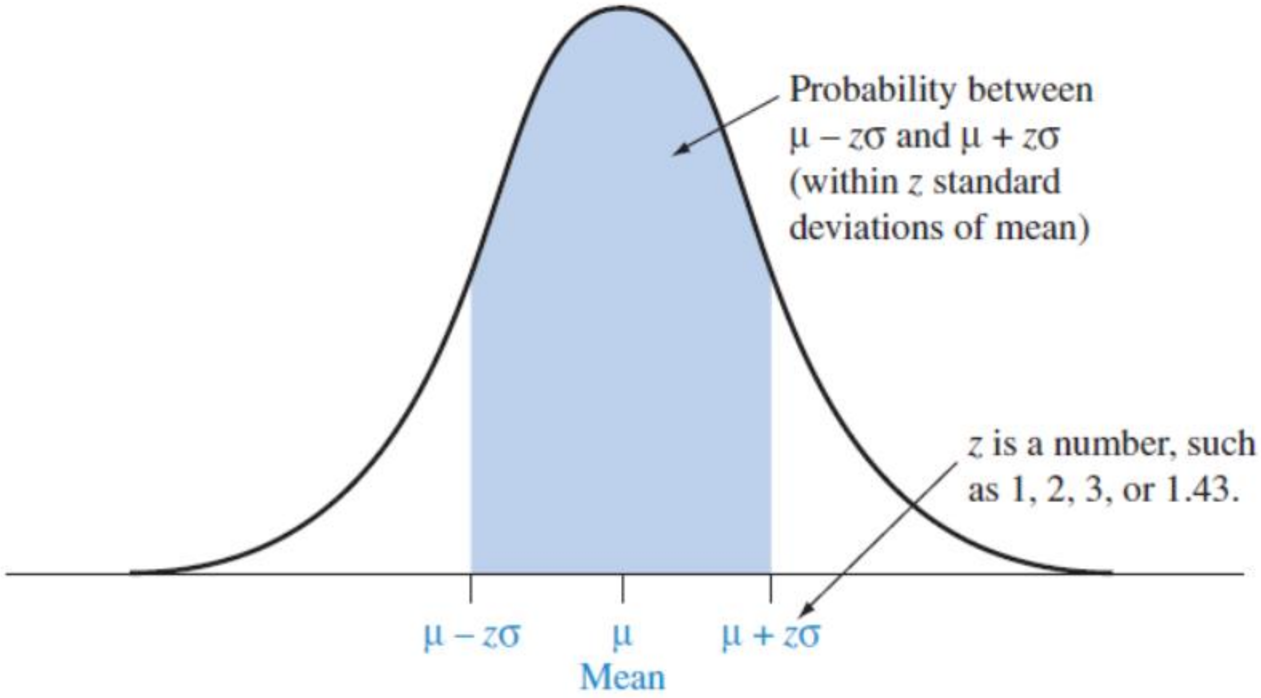
Probability

Normal Distribution – How to use z table

Prof. Dr. Mostafa Elhosseini

https://youtube.com/drmelhosseini

Revise



- <u>For each fixed number z</u>, the probability within z standard deviations of the mean is the area under the normal curve between $\mu - z\sigma$ and $\mu + z\sigma$

Revise

The **proportion** of a normal population that is **within a given number of standard deviations of the mean** is the same for any normal population.

- For this reason, **when dealing with normal populations**, we often **convert from** the units in which the population items were originally measured **to standard units**.

Example - 01

Assume that the heights in a population of women follow the normal curve with mean $\boldsymbol{\mu = 64}$ inches and standard deviation $\boldsymbol{\sigma = 3}$ inches. The heights of two randomly chosen women are **67 inches** and **62 inches**.

- **Convert these heights to standard units**.
- A height of 67 inches is 3 inches more than the mean of 64, and 3 inches is equal to one standard deviation. So 67 inches is one standard deviation above the mean and is thus equivalent to one standard unit. A height of 62 inches is 0.67 standard deviations below the mean, so 62 inches is equivalent to −0.67 standard units

z-score

- if x is an item sampled from a normal population with mean μ and variance σ^2, the standard unit equivalent of x is the number z, where

$$\boldsymbol{z = \frac{x - \mu}{\sigma}}$$

- The number **z** is sometimes called the "**z-score**" of x. The z-score is an item sampled from a normal population with **mean 0** and **standard deviation 1**.
- This normal population is called the **standard normal population**

Example - 02

Aluminum sheets used to make beverage cans have thicknesses (in thousandths of an inch) that are normally distributed with **mean 10** and **standard deviation 1.3**. A particular sheet is 10.8 thousandths of an inch thick.

- Find the z-score.
- The quantity 10.8 is an observation from a normal population with mean $\mu = 10$ and standard deviation $\sigma = 1.3$. Therefore.

$$\boldsymbol{z = \frac{10.8 - 10}{1.3} = 0.62}$$

Standard Normal Distribution

- The normal distribution with parameter values $\mu = 0$and $\sigma = 1$ is called the **standard normal distribution**.
- A random variable having a standard normal distribution is called a standard normal random variable and will be denoted by **Z**.
- The pdf of Z is

$$f(z;0,1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad -\infty < z < \infty$$

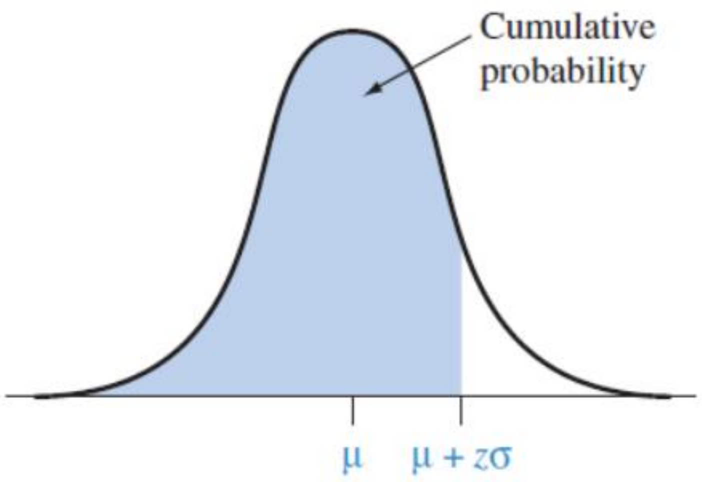
- The **standard normal distribution** does **not** frequently **serve** as a model for a naturally arising population. Instead, it is a **reference distribution** from which information about other normal distributions can be **obtained**

Standard Normal Distribution

- Areas under the **standard** normal curve (**mean 0**, **variance 1**) have been extensively **tabulated**.
- A **typical** such table, called a standard normal table, or **z table**
- To **find areas under a normal curve** with a different mean and variance, we **convert to standard** units and **use the z table**

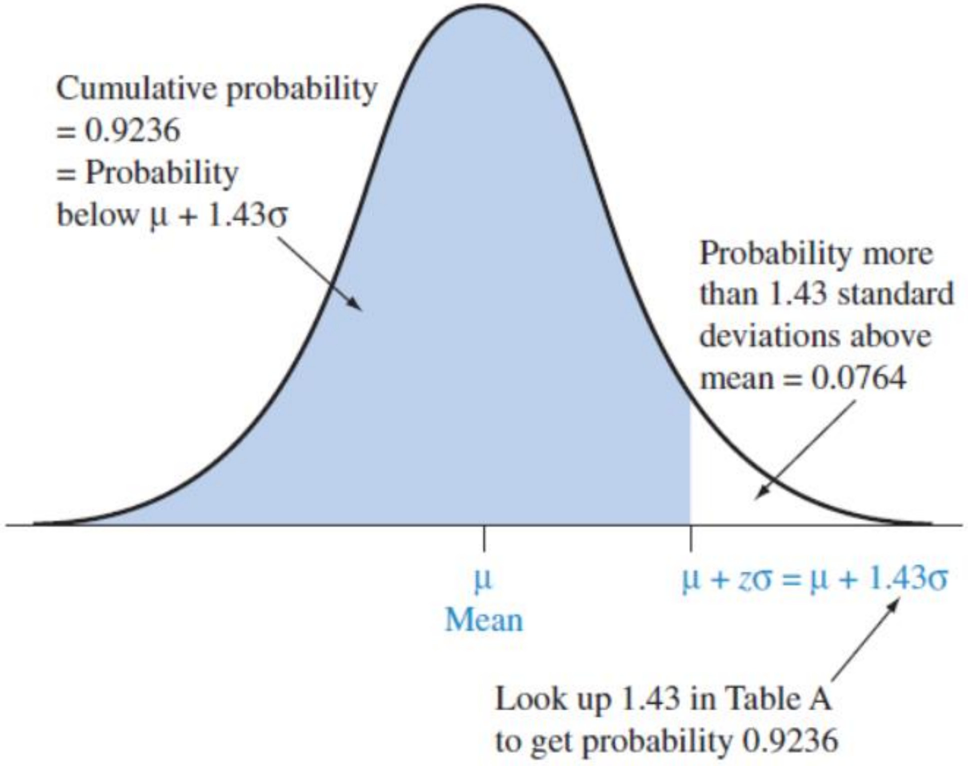
How to use z-table?

- It tabulates the normal cumulative probability, the probability of falling below the point $\mu + z\sigma$.



Second Decimal Place of z										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
…										
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9139	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

How to use z-table?

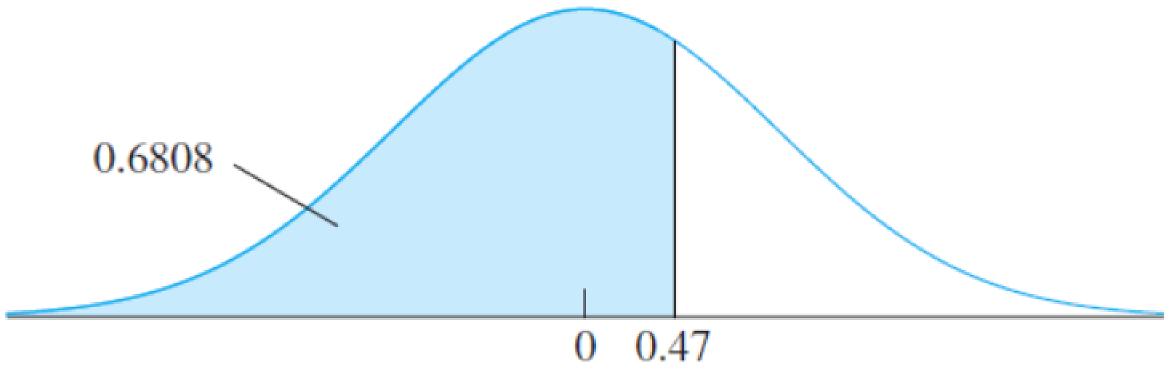


Second Decimal Place of *z*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
...										
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9139	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441

Example - 03

- Find the area under the normal curve to the left of z = 0.47



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

Prob. 01

Suppose the current measurements in a strip of wire are assumed to follow a **normal distribution** with a **mean of 10** milliamperes and a **variance of 4**.

- What is the probability that a measurement will **exceed 13 milliamperes**?