Probability

|Basic Ideas

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Intro…

- The development of the theory of probability was financed by seventeenth-century **gamblers**, who hired some of the leading mathematicians of the day to **calculate the correct odds for certain games of chance**.
- **Later**, people realized that **scientific processes involve chance as well**, and since then the **methods of probability** have been used to **study the physical world**

Intro…

- The term **probability** refers to the **study of randomness and uncertainty**
- It is **likely** that the **Dow-Jones** average will increase by the end of the year
- There will **probably** be at least one section of that course offered next year
- It is **expected** that at least 20,000 concert tickets will be sold

Terminology

- To make a systematic study of probability, we need some **terminology**

Experiment

- An **experiment** is a process that results in an outcome that cannot be predicted in advance with certainty.
  - Tossing a **coin**,
  - Rolling a **die**,
  - Measuring the **diameter of a bolt**,
  - **Weighing** the contents of a box of cereal,
  - **Selecting a card** or cards from a deck,
  - **Weighing a loaf of bread**,
  - Obtaining **blood types** from a group of individuals, and
  - **Measuring** the compressive strengths of different steel beams are all examples of experiments

Trial

- **Performance of an experiment** is called a **trial** of the experiment

Outcome

- An **observed result**, on a trial of the experiment, is called an **outcome**

Sample Space

- The **set of all possible outcomes** of an experiment is called the **sample space** for the experiment.
- Denoted by S

Example – Finite Sample Space

- For **tossing a coin**, we can use the set **{Heads, Tails}** as the sample space.
- For **rolling a six-sided die**, we can use the set **{1, 2, 3, 4, 5, 6}**. These sample spaces are **finite**.
- Examining a **single fuse** to see whether it is defective. The sample space for this experiment can be abbreviated as $\boldsymbol{S} = \{\boldsymbol{N}, \boldsymbol{D}\}$, where **N** represents not defective, **D** represents defective

Example – Finite Sample Space

- **Two gas stations** are located at a certain intersection. **Each one has six gas pumps**. Consider the **experiment** in which the **number of pumps in use at a particular time** of day is determined for each of the stations. An experimental outcome specifies how many pumps are in use at the first station and how many are in use at the second one. **One possible outcome is (2, 2)**, **another is (4, 1)**, and yet another is (1, 4). The **49 outcomes** in S are displayed in the accompanying table

Example – Finite Sample Space

		Second Station						
		0	**1**	**2**	**3**	**4**	**5**	**6**
	0	(0, 0)	(0, 1)	(0, 2)	(0, 3)	(0, 4)	(0, 5)	(0, 6)
	1	(1, 0)	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
First Station	**3**	(3, 0)	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 0)	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 0)	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 0)	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example – Infinite Sample Space

- Some experiments have sample spaces with an **infinite number of outcomes**
- **Punch with diameter 10 mm punches holes in sheet metal**. Because of variations in the angle of the punch and slight movements in the sheet metal, the diameters of the holes vary between 10.0 and 10.2 mm
- Reasonable sample space is the **interval (10.0, 10.2)**, or in set notation, **$\{x \mid 10.0 < x < 10.2\}$**

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Event

- In our study of probability, we will be **interested** not only in the individual outcomes of S but also in **various collections of outcomes from S**.
- A **subset of a sample space** is called an **event**
- When discussing experiments, we are often **interested in a particular subset of outcomes**.
- For example, we might be interested in the probability that a **die comes up an even number**. The sample space for the experiment is **{1, 2, 3, 4, 5, 6}**, and coming up even corresponds to the subset **{2, 4, 6}**.

Event

- In the hole punch example, we might be interested in the probability that a hole has a diameter less than 10.1 mm.
- This correspond to the subset $\{x \mid 10.0 < x < 10.1\}$
- A **given event is said to have occurred** if the outcome of the experiment is one of the outcomes in the event. For example, if a die comes up 2, the events {2, 4, 6} and {1, 2, 3} have both occurred, along with every other event that contains the outcome "2."

Example

- An electrical engineer has on hand **two boxes of resistors**, with **four** resistors in each box. The resistors in the first box are labeled **10 Ω** (ohms), but in fact their resistances are **9, 10, 11, and 12 Ω**. The resistors in the second box are labeled **20 Ω**, but in fact their resistances are **18, 19, 20, and 21 Ω**. The engineer chooses one resistor from each box and determines the resistance of each.
- Let **A** be the event that the **first resistor has a resistance greater than 10**, let **B** be the event that the **second resistor has a resistance less than 19**, and let **C** be the event that the **sum of the resistances is equal to 28**.
- Find a **sample space** for this experiment, and specify the subsets corresponding to the events **A, B, and C**.

Example

$$\mathcal{S} = \{(9, 18), (9, 19), (9, 20), (9, 21), (10, 18), (10, 19), (10, 20), (10, 21), (11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$

The events A, B, and C are given by

$$A = \{(11, 18), (11, 19), (11, 20), (11, 21), (12, 18), (12, 19), (12, 20), (12, 21)\}$$
$$B = \{(9, 18), (10, 18), (11, 18), (12, 18)\}$$
$$C = \{(9, 19), (10, 18)\}$$

Example

When the number of pumps in use at each of two six-pump gas stations is observed, there are 49 possible outcomes, so there are 49 simple events: $E_1 = \{(0, 0)\}$, $E_2 = \{(0, 1)\}, \ldots, E_{49} = \{(6, 6)\}$. Examples of compound events are

$A = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ = the event that the number of pumps in use is the same for both stations

$B = \{(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)\}$ = the event that the total number of pumps in use is four

$C = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ = the event that at most one pump is in use at each station ■

References

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