Probability

Random Variable

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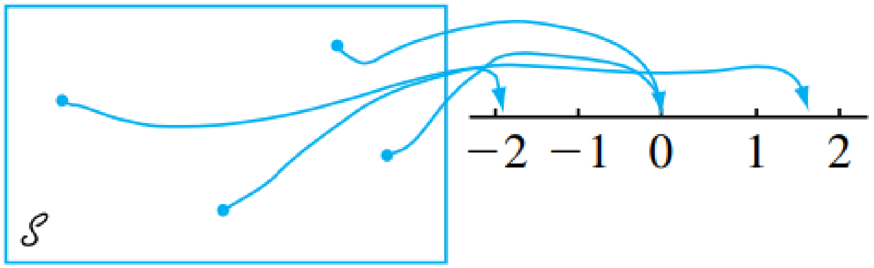
Intro…

- In many situations, it is desirable to **assign a numerical value to each outcome of an experiment**.
- Such an assignment is called a **random variable (rule of association)**
  - The **number** among the sample of ten components that **fail to last 1000 hours**
  - The **total weight** of baggage for a sample of 25 airline passengers



Random Variable

- Such a **rule of association** is called a **random variable**
  - **Variable** because different numerical values are possible and
  - **Random** because the observed value depends on which of the possible experimental outcomes results
- Random variables are customarily denoted by **uppercase letters**, such as $\boldsymbol{X}$ and $\boldsymbol{Y}$, near the end of our alphabet
- we will now use **lowercase letters** to **represent some particular value** of the corresponding random variable.
- The notation $X(s) = x$ means that x is the value associated with the outcome s by the random variable X

Example - 01

- Suppose that an electrical engineer has on hand **six resistors. Three** of them are **labeled $\mathbf{10\ \Omega}$** and the **other three** are labeled $\mathbf{20\ \Omega}$. The engineer **wants to connect a $\mathbf{10\ \Omega}$ resistor and a $\mathbf{20\ \Omega}$ resistor** in series, to **create a resistance of $\mathbf{30\ \Omega}$**.
- Now suppose that in fact the **three resistors labeled $\mathbf{10\ \Omega}$** have actual resistances of **9,10, and $\mathbf{11\ \Omega}$**, and that the **three resistors labeled $\mathbf{20\ \Omega}$** have actual resistances of **19, 20, and $\mathbf{21\ \Omega}$**.
- The process of **selecting one resistor of each type is an experiment** whose **sample space consists of nine** equally likely outcomes

Example - 01

- Now **what is important** to the engineer in this experiment is the **sum of the two resistances, rather than their individual values**.
- Therefore, we **assign to each outcome a number equal to the sum of the two resistances selected**. This assignment, represented by the letter X

Outcome	Probability
(9, 19)	1/9
(9, 20)	1/9
(9, 21)	1/9
(10, 19)	1/9
(10, 20)	1/9
(10, 21)	1/9
(11, 19)	1/9
(11, 20)	1/9
(11, 21)	1/9

Example - 01

- A **random variable** assigns a numerical value to each outcome in a sample space.
- We can compute probabilities for random variables in an obvious way.
- The **event X = 29** corresponds to the event **{(9, 20), (10, 19)}** of the sample space.
- Therefore $\boldsymbol{P(X = 29)} = P(\{(9,20),(10,19)\}) = \mathbf{2/9}$.

Outcome	X	Probability
(9, 19)	28	1/9
(9, 20)	29	1/9
(9, 21)	30	1/9
(10, 19)	29	1/9
(10, 20)	30	1/9
(10, 21)	31	1/9
(11, 19)	30	1/9
(11, 20)	31	1/9
(11, 21)	32	1/9

Example - 02

- List the possible values of the random variable X, and find the probability of each of them.

Outcome	X	Probability
(9, 19)	28	1/9
(9, 20)	29	1/9
(9, 21)	30	1/9
(10, 19)	29	1/9
(10, 20)	30	1/9
(10, 21)	31	1/9
(11, 19)	30	1/9
(11, 20)	31	1/9
(11, 21)	32	1/9

Example - 02

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(10, 19)	29	1/9
(10, 20)	30	1/9
(10, 21)	31	1/9
(11, 19)	30	1/9
(11, 20)	31	1/9
(11, 21)	32	1/9

x	$P(X = x)$
28	1/9
29	2/9
30	3/9
31	2/9
32	1/9

Example - 03

- When a student attempts to **log on to a computer time-sharing system**, **either all ports are busy (F)**, in which case the student will **fail to obtain access**, or else there is **at least one port free (S)**, in which case the student will be successful in accessing the system. With $\mathcal{S}\{\boldsymbol{S}, \boldsymbol{F}\}$, define random variable X by $X(S) = 1, (F) = 0$
- The random X indicates whether (1) or not (0) the student can log on

Example - 04

- Consider the experiment in which a **telephone number in a certain area code is dialed using a random number dialer** (such devices are used extensively by **polling** organizations), and define random variable Y by

- $Y = \begin{cases} 1 & \textit{if the selected number is unlisted} \\ 0 & \textit{if the selected number is listed in the directory} \end{cases}$

- if 5282966 appears in the telephone directory, then $Y(\mathbf{5282966}) = \mathbf{0}$, whereas $Y(\mathbf{7727350}) = \mathbf{1}$ tells us that the number 7727350 is unlisted
- Any **random variable** whose **only possible values** are **0 and 1** is called a Bernoulli random variable

Example - 05

- In an experiment in which the **number of pumps in use at each of two gas stations was determined**. Define random variable X, Y, and U by
  - X the total number of pumps in use at the two stations
  - Y the difference between the number of pumps in use at station 1 and the number in use at station 2
  - U the maximum of the numbers of pumps in use at the two stations
- If this experiment is performed and $s(2, 3)$ results
  - $X(s(2,3)) = 5$
  - $Y(s(2,3)) = -1$
  - $U(s(2,3)) = 3$

Example - 06

- Each of the random variables of the above examples can assume only a finite number of possible values. This need not be the case
- the experiment in which batteries were examined until a good one (S) was obtained. The sample space was $\mathcal{S}$ $\{S, FS, FFS, \ldots\}$.
  Define random variable X by

  X : the number of batteries examined before the experiment terminates
  - $X(S) = 1$
  - $X(FS) = 2$
  - $X(FFS) = 3$
  - $X(FFFFFFS) = 7$
- Any positive integer is a possible value of X, so the set of possible values is infinite

Example - 07

- The experiment of tossing a coin twice. Let X represent number of heads appearing
  - X is a random variable with values 0, 1, 2
  - $X(T, T) = 0$
  - $X(H, T) = 1$

Example - 08

- Suppose that in some random fashion, a location (latitude and longitude) in the continental United States is selected. **Define random $\boldsymbol{Y}$ by**
  Y the height above sea level at the selected location
- $Y((39° \, (50)` \; N, 98° \, (35)` \; W)) = "1748.26 \text{ ft}"$
- There are an **infinite number** of numbers in this interval

Example - 09

- **Computer chips** often contain surface **imperfections**. For a certain type of computer chip, **9% contain no imperfections**, **22%** contain **1** imperfection, **26%** contain **2** imperfections, **20%** contain **3** imperfections, **12%** contain **4** imperfections, and the remaining **11%** contain **5** imperfections. Let Y represent the number of imperfections in a randomly chosen chip.
- What are the possible values for Y? Is Y discrete or continuous? Find $P(Y = y)$ for each possible value y.

Example - 10

- A certain type of **magnetic disk** must function in an environment where it is **exposed to corrosive gases**. It is known that **10%** of all such disks have **lifetimes less than or equal to 100 hours, 50%** have lifetimes **greater than 100 hours but less than or equal to 500 hours**, and **40%** have lifetimes **greater than 500 hours**.
- Let Z represent the number of hours in the lifetime of a randomly chosen disk. Is Z **continuous or discrete**? Find $P(Z \leq 500)$. Can we compute all the probabilities for Z? Explain.

Discrete – Continuous Random Variable

- When the probabilities pertaining to a random variable are known, we usually do not think about the sample space; we just focus on the probabilities
- There are two important types of random variables, **discrete and continuous.**
  - A **discrete random variable** is one whose possible values form a **discrete set**.
  - The possible values of a **continuous random variable** always contain an **interval**, that is, all the points between some two numbers.

Discrete – Continuous Random Variable

- Discrete
  - Finite set
  - Infinite “But counted”
- continuous
  - Interval
  - $P(X=c) = 0$ for any possible value of c

Probability Mass Function

- **The number of flaws in a 1-inch length of copper wire** manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**. Let X be the number of flaws in a randomly selected piece of wire. Then
  - $P(X = 0) = 0.48$ $P(X = 1) = 0.39$
  - $P(X = 2) = 0.12$ $P(X = 3) = 0.01$
- The list of possible values **0, 1, 2, 3**, **along** with the **probabilities** for each, provide a **complete description of the population** from which X is drawn. This description has a name—the **probability mass function**

References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi Colorado School of Mines
- Khan Academy