# Probability

## Random Variable [ Cumulative Distribution Function ]

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# Cumulative Distribution Function

- The **probability mass function** specifies the **probability that a random variable is equal to a given value**.
- The **cumulative distribution function** specifies the **probability that a random variable is less than or equal to a given value**
- The cumulative distribution function of the random variable X is the function $F(x) = P(X \leq x)$.

# Example - 01

- The number of flaws in a 1-inch length of copper wire manufactured by a certain process varies from wire to wire. Overall, 48% of the wires produced have no flaws, 39% have one flaw, 12% have two flaws, and 1% have three flaws. Let $X$ be the number of flaws in a randomly selected piece of wire. Then

  $P(X = 0) = 0.48$ $\qquad$ $P(X = 1) = 0.39$

  $P(X = 2) = 0.12$ $\qquad$ $P(X = 3) = 0.01$

- Let **F(x)** denote the **cumulative distribution function** of the random variable X that represents the number of flaws in a randomly chosen wire. Find F(2). Find F(1.5).

# Example - 01

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- $F(2) = P(X \leq 2) = 0.99$
- $F(1.5) = P(X \leq 1.5) = 0.87$

# Note…

- In general, for any discrete random variable X, the **cumulative distribution function F(x) can be computed by summing the probabilities of all the possible values of X that are less than or equal to x**.
- Note that **F(x) is defined for any number x**, not just for the possible values of X

# Example - 02

- Plot the cumulative distribution function $F(x)$ of the random variable $X$ that represents the number of flaws in a randomly chosen wire.

# Example - 02

- Plot the cumulative distribution function F(x) of the random variable X that represents the number of flaws in a randomly chosen wire.

First we compute F(x) for each of the possible values of X, which are 0, 1, 2, and 3.

- $F(0) = P(X \leq 0) = 0.48$
- $F(1) = P(X \leq 1) = 0.48 + 0.39 = 0.87$
- $F(2) = P(X \leq 2) = 0.48 + 0.39 + 0.12 = 0.99$
- $F(3) = P(X \leq 3) = 0.48 + 0.39 + 0.12 + 0.01 = 1$

# Example - 02

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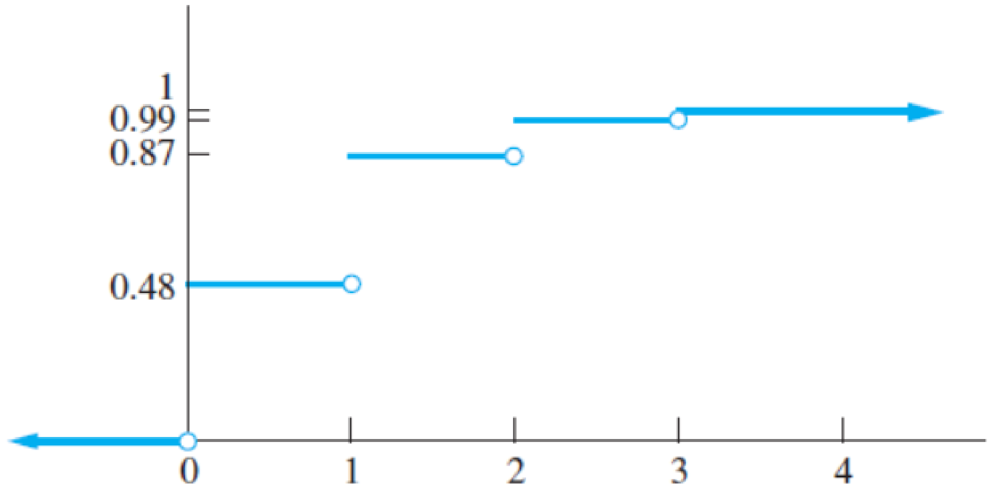
- $F(1) = P(X \leq 1) = 0.48 + 0.39 = 0.87$
- $F(2) = P(X \leq 2) = 0.48 + 0.39 + 0.12 = 0.99$
- $F(3) = P(X \leq 3) = 0.48 + 0.39 + 0.12 + 0.01 = 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.48 & 0 \leq x < 1 \\ 0.87 & 1 \leq x < 2 \\ 0.99 & 2 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$



# Note…

- For a **discrete random variable**, the graph of **F(x)** consists of a series of **horizontal lin**es (called "**steps**") with **jumps at each of the possible values of X**.
- Note that the **size of the jump at any point x is equal to the value of the probability mass function** $\boldsymbol{p(x) = P(X = x)}$.

# References


- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
  Colorado School of Mines
- Khan Academy