

Probability

Random Variable - Examples

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| Example - 01

- When a student attempts to **log on to a computer time-sharing system**, **either all ports are busy (F)**, in which case the student will **fail to obtain access**, or else there is **at least one port free (S)**, in which case the student will be successful in accessing the system. With $\mathcal{S}\{S, F\}$, define random variable X by $X(S) = 1, (F) = 0$
- The random X indicates whether (1) or not (0) the student can log on

| Example - 02

- Consider the experiment in which a **telephone number in a certain area code is dialed using a random number dialer** (such devices are used extensively by **polling** organizations), and define random variable Y by
- $$Y = \begin{cases} 1 & \text{if the selected number is unlisted} \\ 0 & \text{if the selected number is listed in the directory} \end{cases}$$
- if 5282966 appears in the telephone directory, then $Y(\mathbf{5282966}) = \mathbf{0}$, whereas $Y(\mathbf{7727350}) = \mathbf{1}$ tells us that the number 7727350 is unlisted
- Any **random variable** whose **only possible values** are **0 and 1** is called a Bernoulli random variable

| Example - 03

- In an experiment in which the **number of pumps in use at each of two gas stations was determined**. Define random variable X , Y , and U by
 - X the total number of pumps in use at the two stations
 - Y the difference between the number of pumps in use at station 1 and the number in use at station 2
 - U the maximum of the numbers of pumps in use at the two stations
- If this experiment is performed and $s(2, 3)$ results
 - $X(s(2,3)) = 5$
 - $Y(s(2,3)) = -1$
 - $U(s(2,3)) = 3$

| Example - 04

- Each of the random variables of the above examples can assume only a finite number of possible values. This need not be the case
- the experiment in which batteries were examined until a good one (S) was obtained. The sample space was $\mathcal{S} \{S, FS, FFS, \dots\}$.
Define random variable X by
 - X : the number of batteries examined before the experiment terminates
 - $X(S) = 1$
 - $X(FS) = 2$
 - $X(FFS) = 3$
 - $X(FFFFFFFS) = 7$
- Any positive integer is a possible value of X , so the set of possible values is infinite

| Example - 05

- The experiment of tossing a coin twice. Let X represent number of heads appearing
 - X is a random variable with values 0, 1, 2
 - $X(T, T) = 0$
 - $X(H, T) = 1$

| Example - 06

- Suppose that in some random fashion, a location (latitude and longitude) in the continental United States is selected. **Define random Y by**
 Y the height above sea level at the selected location
- $Y((39^\circ (50)' N, 98^\circ (35)' W)) = "1748.26 \text{ ft}"$
- There are an **infinite number** of numbers in this interval

| Example - 07

- **Computer chips** often contain surface **imperfections**. For a certain type of computer chip, **9% contain no imperfections**, **22% contain 1 imperfection**, **26% contain 2 imperfections**, **20% contain 3 imperfections**, **12% contain 4 imperfections**, and the remaining **11% contain 5 imperfections**. Let Y represent the number of imperfections in a randomly chosen chip.
- What are the possible values for Y ? Is Y discrete or continuous? Find $P(Y = y)$ for each possible value y .

| Example - 10

- A certain type of **magnetic disk** must function in an environment where it is **exposed to corrosive gases**. It is known that **10%** of all such disks have **lifetimes less than or equal to 100 hours**, **50%** have lifetimes **greater than 100 hours but less than or equal to 500 hours**, and **40%** have lifetimes **greater than 500 hours**.
- Let Z represent the number of hours in the lifetime of a randomly chosen disk. Is Z **continuous or discrete**? Find $P(Z \leq 500)$. Can we compute all the probabilities for Z ? Explain.

| Discrete – Continuous Random Variable

- When the probabilities pertaining to a random variable are known, we usually do not think about the sample space; we just focus on the probabilities
- There are two important types of random variables, **discrete** and **continuous**.
 - A **discrete random variable** is one whose possible values form a **discrete set**.
 - The possible values of a **continuous random variable** always contain an **interval**, that is, all the points between some two numbers.

| Discrete – Continuous Random Variable

- Discrete
 - Finite set
 - Infinite “But counted”
- continuous
 - Interval
 - $P(X=c) = 0$ for any possible value of c

| Probability Mass Function

- **The number of flaws in a 1-inch length of copper wire** manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**. Let X be the number of flaws in a randomly selected piece of wire. Then
 - $P(X = 0) = 0.48$ $P(X = 1) = 0.39$
 - $P(X = 2) = 0.12$ $P(X = 3) = 0.01$
- The list of possible values **0, 1, 2, 3**, along with the **probabilities** for each, provide a **complete description of the population** from which X is drawn. This description has a name—the **probability mass function**

| References

- Statistics for Engineers and Scientists, Fourth Edition, William Navidi
Colorado School of Mines
- Khan Academy