Probability

Probability Mass Function |Random Variable

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Intro

- It is common for the possible values of a discrete random variable to be a set of integers.
- For any discrete random variable, if we specify the **list of its possible values along with the probability** that the random variable takes on each of these values, then we have **completely described** the **population** from which the random variable is sampled

Probability Mass Function

- Suppose, for example, that a business has just purchased **four laser printers**, and let X be the number among these that **require service during the warranty period**.
  Possible X values are then 0, 1, 2, 3, and 4.
- The probability distribution will tell us how the probability of 1 is subdivided among these five possible values
  - $p(0)$ = the probability of the X value $0 \;=\; p(X=0)$
  - $p(1)$ = the probability of the X value $1 \;=\; p(X=1)$
  - in general, $p(x)$ will denote the probability assigned to the value x

Probability Mass Function

- **The number of flaws in a 1-inch length of copper wire** manufactured by a certain process varies from wire to wire. Overall, **48%** of the wires produced have **no flaws**, **39%** have **one flaw**, **12%** have **two flaws**, and **1%** have **three flaws**. Let X be the number of flaws in a randomly selected piece of wire. Then
  - $P(X = 0) = 0.48 \quad P(X = 1) = 0.39$
  - $P(X = 2) = 0.12 \quad P(X = 3) = 0.01$
- The list of possible values **0, 1, 2, 3**, **along** with the **probabilities** for each, provide a **complete description of the population** from which X is drawn. This description has a name—the **probability mass function**

Probability Mass Function

- The **probability mass function** of a discrete random variable X is the function $\boldsymbol{p(x) = P(X = x)}$.
- The probability mass function is sometimes called the **probability distribution**.
- The **probability distribution** of X says **how the total probability of 1 is distributed among (allocated to) the various possible X values**

Probability Mass Function

- Note that if the **values of the probability mass function are added over all the possible values of X, the sum is equal to 1**.
- This is **true** for any probability mass function.
- The reason is that **summing the values of a probability mass function** over all the possible values of the corresponding random variable produces the **probability that the random variable is equal to one of its possible values**, and this probability is always **equal to 1**

Example - 01

- A certain gas station has **six pumps**. Let X denote the **number of pumps that are in use** at a particular time of day. Suppose that the **probability distribution of X** is as given in the following table; the first row of the table lists the possible X values and the second row gives the probability of each such value.

x	0	1	2	3	4	5	6
$p(x)$	.05	.10	.15	.25	.20	.15	.10

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Example - 01

We can now use elementary probability properties to calculate other probabilities of interest

- the probability that at most 2 pumps are in use is

$$p(x \le 2) = p(x = 0 \; or \; 1 \; or \; 2) = p(0) + p(1) + p(2) = 0.3$$

- Since the event at least 3 pumps are in use is complementary to at most 2 pumps are in use,

$$p(x \ge 3) = 1 - \; p(x \le 2) = 1 - 0.3 = 0.7$$

- which can, of course, also be obtained by adding together probabilities for the values, 3, 4, 5, and 6
- The probability that between 2 and 5 pumps inclusive are in use ?

$$P(2 \le x \le 5)$$

Example - 02

- Six lots of components are ready to be shipped by a certain supplier. The number of defective components in each lot is as follows:

Lot	1	2	3	4	5	6
Number of defectives	0	2	0	1	2	0

- **One of these lots is to be randomly selected** for shipment to a particular customer. Let X be the number of defectives in the selected lot. The three possible X values are 0, 1, and 2
- $p(0) = p(X = 0) = p(lot\ 1\ or\ 3\ or\ 6\ is\ sent) = 3/6 = 0.5$
- $p(1) = p(X = 1) = p(lot\ 4\ is\ sent) = 1/6 = 0.167$
- $p(2) = p(X = 2) = p(lot\ 2\ or\ 5\ is\ sent) = 2/6 = 0.333$

Example - 03

- The experiment of **tossing a coin twice**. Let X represent number of heads appearing
  - X is a random variable with values 0, 1, 2
  - $p(X = 0) = p(TT) = 1/4$
  - $p(X = 1) = p(HT, TH) = 2/4$
  - $p(X = 2) = p(HH) = 1/4$

Example - 04

- Considering the experiment of tossing two dice. The random variable X represents the sum of the two numbers. Find the probability distribution $p(X)$

Probability Mass Function - Graph

- The **probability mass function can be represented by a graph** in which a **vertical line** is drawn **at each of the possible values** of the random variable. The **heights** of the lines are **equal to the probabilities of the corresponding values**.
- The physical interpretation of this graph is that **each line** represents a **mass equal to its height**

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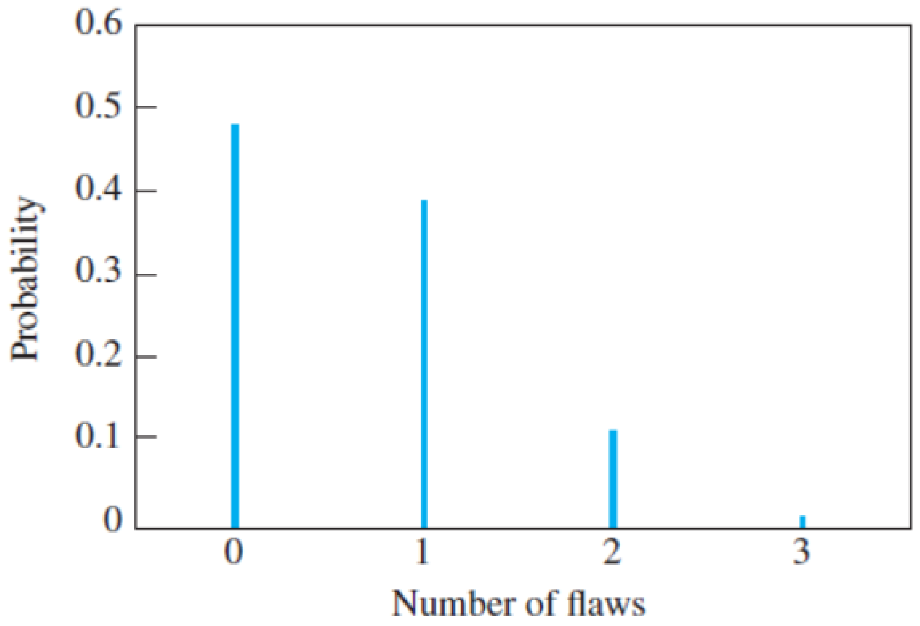

References

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