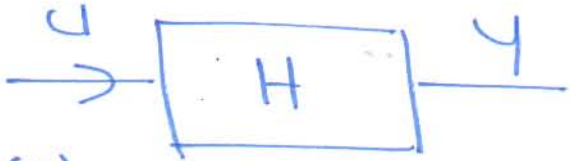
System Response

Once the transfer function has been determined by any of the available methods, we can start to analyze the response



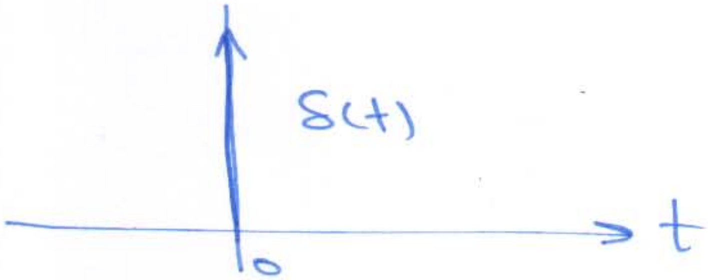
$$H(s) = T.F = \frac{b(s)}{a(s)}$$

Pole: S_ value at which $a(s) = 0$

Zero: S_ value at which $b(s) = 0$

Hint

① impulse input

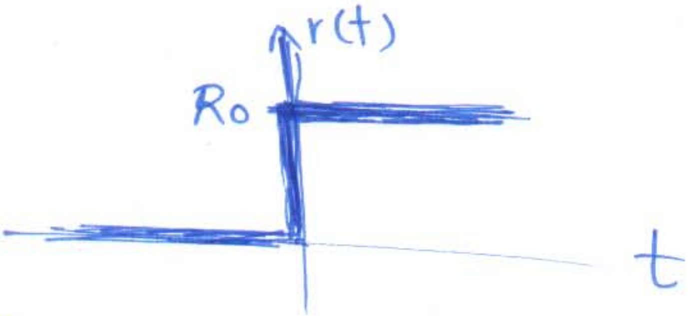


$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\delta(s) = 1$$

② Step input

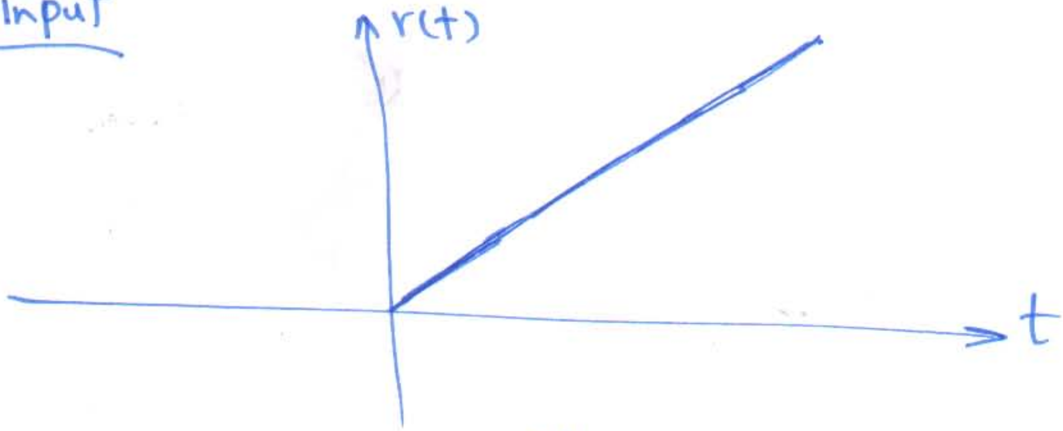


$r(t) = 0 \quad t < 0$

$r(t) = R_o \quad t \geq 0$

$$R(s) = \frac{R_0}{s}$$

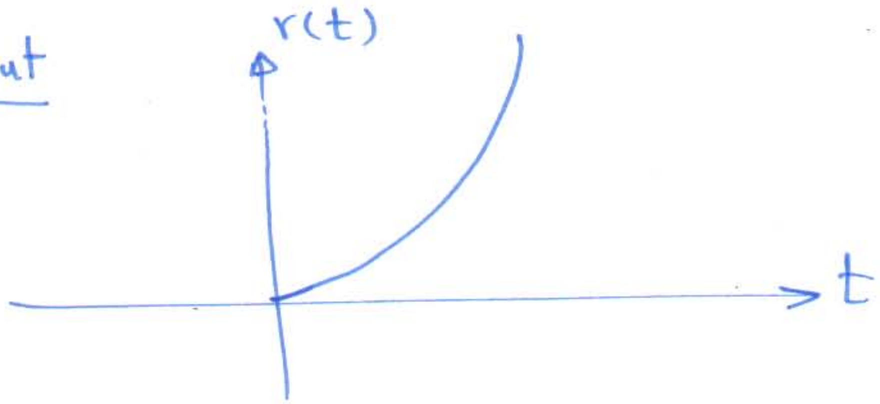
Ramp input



$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$R(s) = \frac{A}{s^2}$$

Parabolic input



$$r(t) = \begin{matrix} At^2 & t \geq 0 \\ 0 & t < 0 \end{matrix}$$

$$R(s) = \frac{2A}{s^3}$$

impulse response = natural freq.

Ex

$$T.f = H(S) = \frac{1}{S+\sigma}$$

impulse response $= h(t) = e^{-\sigma t}$

$\sigma > 0$ ⇒ pole located at $s<0$.
⇒ exponential decays
⇒ impulse response is stable

$\sigma < 0$ ⇒ pole is right of the origin
⇒ exponential term grows with time
⇒ impulse response is unstable

In general

the shapes of the components of the natural responses are determined by the locations of the poles of the t.f

- pole at -2 is faster than the pole at -1
- poles farther to the left in the s-plane are associated with natural signal that decays faster than those associated with poles closer to I-A

- Impulse using Matlab

- Ex

$$H(s) = \frac{2s+1}{s^2+3s+2}$$

```
>> a = [2 1];
>> b = [1 3 2];
>> sys = tf(a,b);
>> impulse(sys);
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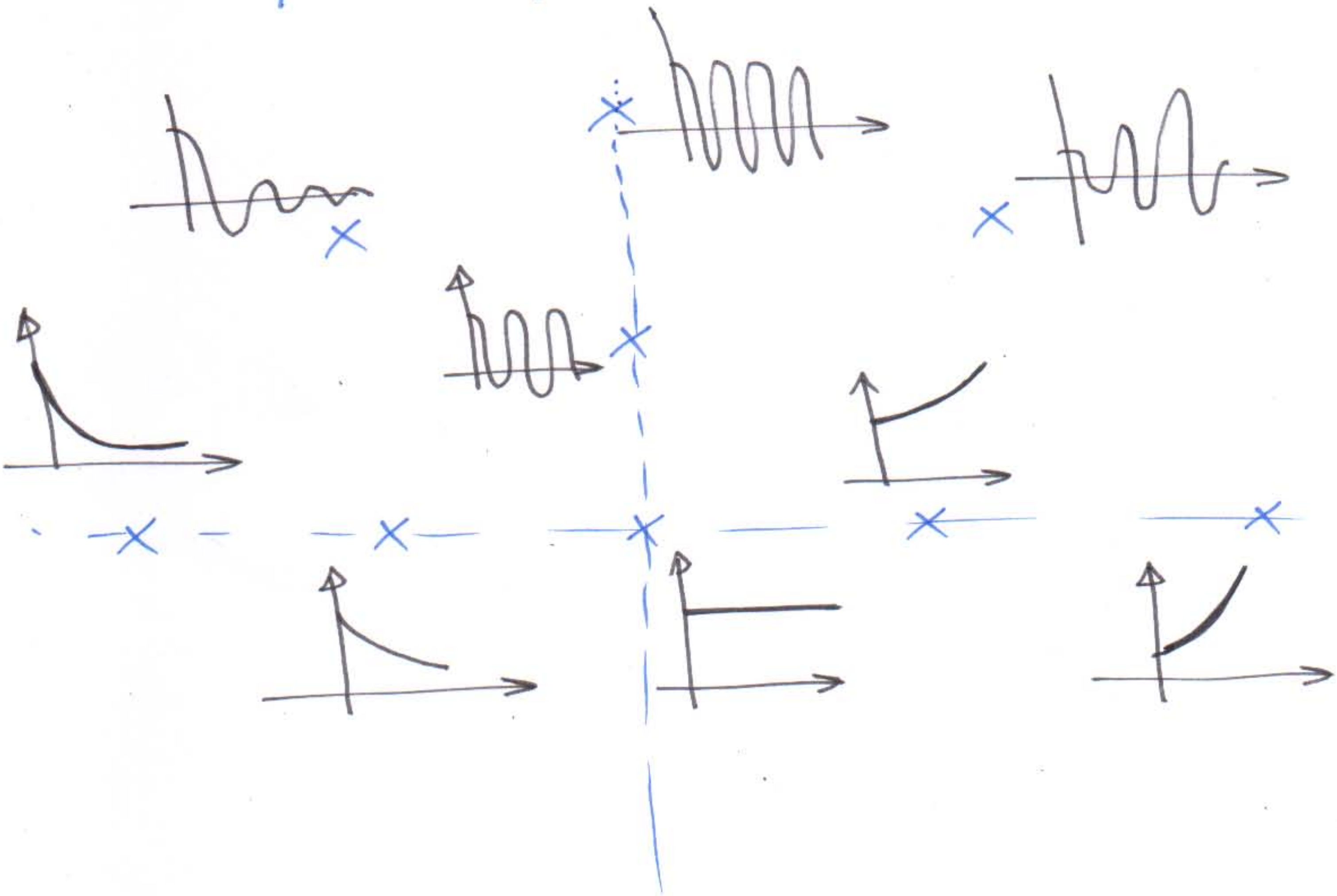


Complex poles can be defined as

$$s = -\sigma \pm j\omega_d$$

Since complex pole come in conjugate pairs, then the denominator will be

$$a(s) = (s+\sigma-j\omega_d)(s+\sigma+j\omega_d)$$
$$= (s+\sigma)^2 + \omega_d^2$$
$$= s^2 + 2\sigma s + \sigma^2 + \omega_d^2$$

when finding the T.f, we typically write the result in rational form

$$\therefore\ H(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

by comparing $a(s)$ with the denominator of $H(s)$

$$\therefore\ \sigma = \xi\omega_n$$

&

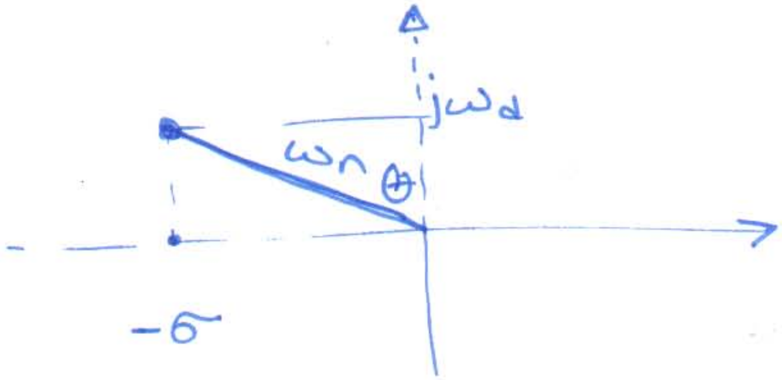
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

ξ : damping ratio

ω_d : damped natural freq.

$\therefore \sigma^2 + \omega_d^2 = \omega_n^2$

$\therefore$ The bold line $= \omega_n$



$$\sin\theta = \frac{\sigma}{\omega_n}$$

$$\sin\theta = \frac{\zeta\omega_n}{\omega_n}$$

$$\therefore \theta = \sin^{-1}\zeta$$

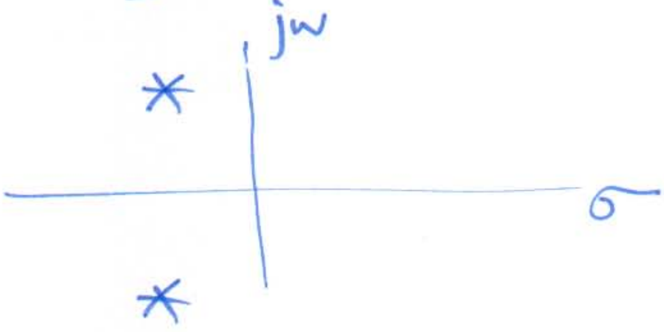
if $\zeta = 0 \Rightarrow \theta = 0$, we have no damping

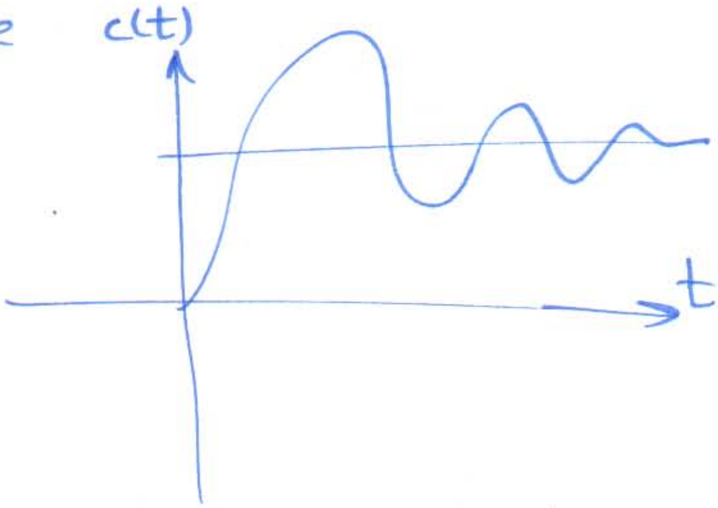
$\therefore \omega_d = \omega_n$ at $\zeta = 0$

$\therefore \omega_d$ = damped natural freq.

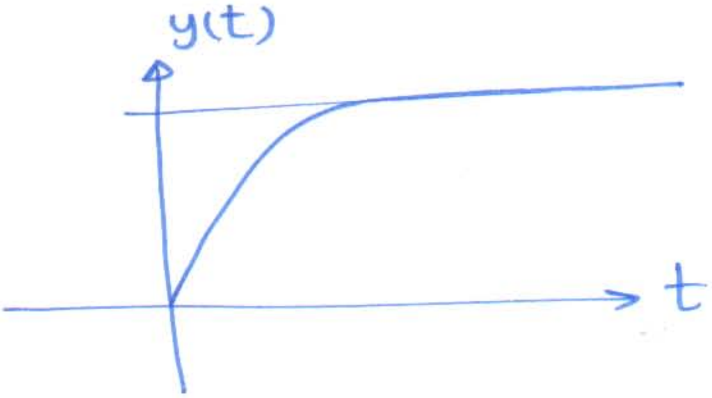
Note $\quad \dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\propto \zeta < 1 \Rightarrow$ underdamped Case

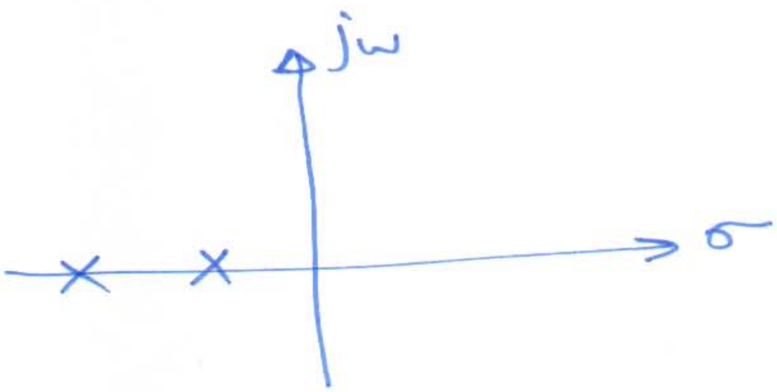


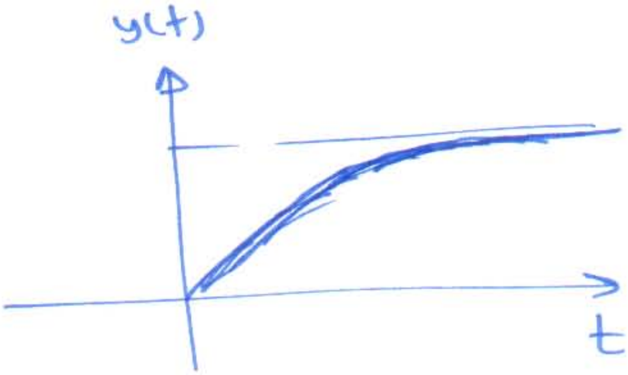


$\zeta = 1 \Rightarrow$ Critically damped



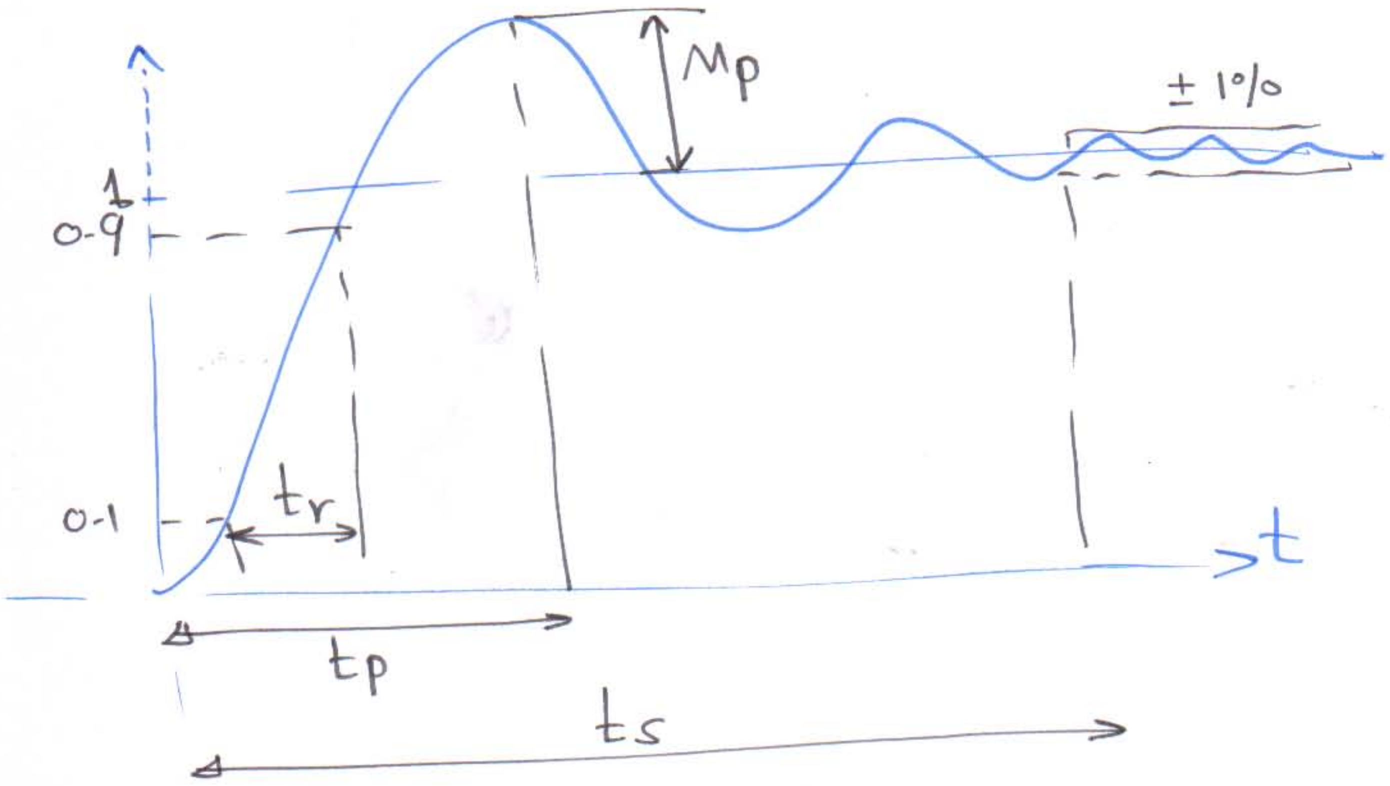
$\zeta > 1 \Rightarrow$ over damped





Time domain specification

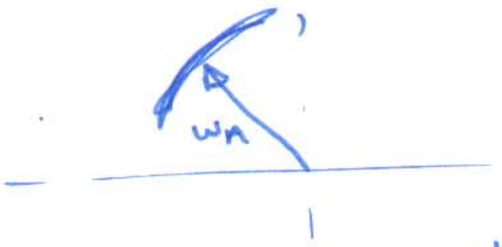
for second-order system, step input, all the curves rise in roughly the same time, if we consider the curve for $\zeta = 0.5$ to be an average



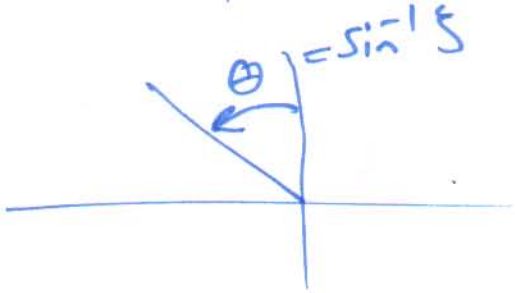
- $t_r \simeq \dfrac{1.8}{\omega_n}$
- $t_p = \dfrac{\pi}{\omega_d}$
- $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \qquad 0<\xi<1$
- $t_s = \dfrac{4.6}{\sigma} = \dfrac{4.6}{\xi\omega_n}$

Design point

- $\omega_n \geq \dfrac{1.8}{t_r}$



- $\xi \geq \xi$ from M_p



- $\sigma \geq \dfrac{4.6}{t_s}$

Ex

Find the allowable regions in the s-plane for the poles of a transfer function if the system response requirement are $t_r \le 0.6$, $M_p \le 10\%$ and $t_s \le 3$

Sol.

$\because t_r = \frac{1.8}{\omega_n}$

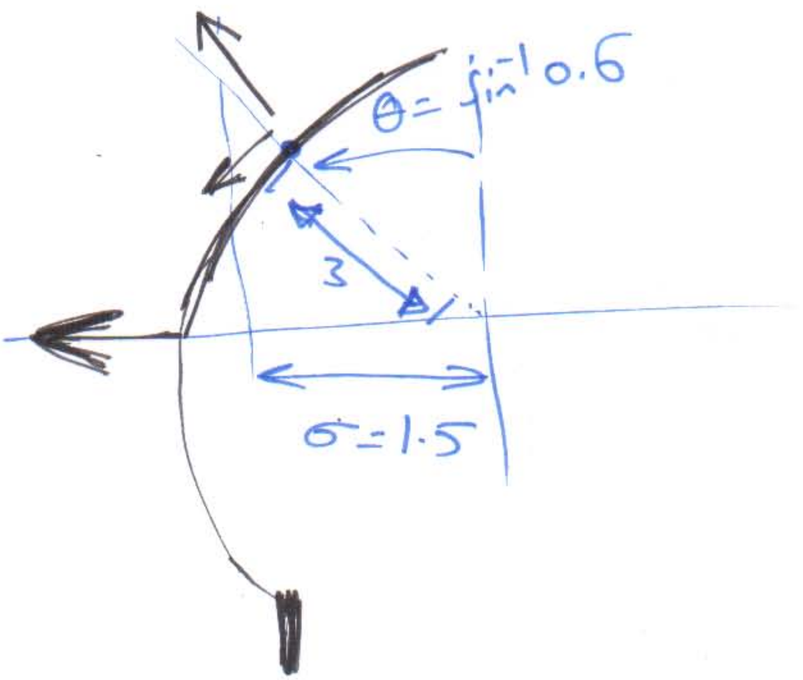
$\therefore \frac{1.8}{\omega_n} \le 0.6 \Rightarrow \omega_n \ge 3$ rad/sec

$\because M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

$\therefore \zeta \ge 0.6$

$\because t_s = \frac{4.6}{\sigma}$

$\frac{4.6}{\sigma} \le 3 \quad \therefore \sigma \ge 1.5$



Prob.

Conside the system shown below

Assume that $\xi = 0.8$

$\omega_n = 12$ rad/sec

Calculate

- delay time
- rise time
- overshoot M_p
- settling time

Ex

$$T(s) = \frac{100}{s^2 + s + 100}$$

Find ω_n, ξ, ω_d

Sol.

$\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\xi\omega_n = 1 \Rightarrow \xi = \frac{1}{20} = 0.05$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 10\sqrt{1-(0.05)^2}$

Prob.

$$T(s) = \frac{3s-49}{s^2 + 3s + 49}$$

Prob.

$$T(s) = \frac{s^2 + 20}{s^2 + 2s + 20}$$

Find

ω_n, ζ, ω_d, t_p, t_s, M_p, t_r