



Form A

FINAL. Exam  
Academic Year 1447 Hijri.- FIRST. Semester

| Exam Information معلومات الامتحان |                                                            |                   |
|-----------------------------------|------------------------------------------------------------|-------------------|
| Course name                       | الاحتمالات والإحصاء الهندسي والاحتمالات والإحصاء للمهندسين | اسم المقرر        |
| Course Code                       | STAT 301 & STAT 305                                        | رمز المقرر        |
| Exam Date                         | 25/12/30                                                   | تاريخ الامتحان    |
| Exam Time                         |                                                            | وقت الامتحان      |
| Exam Duration                     | Two Hours                                                  | مدة الامتحان      |
| Classroom No.                     |                                                            | رقم قاعة الاختبار |
| Instructor Name                   | Prof. Dr. Mostafa Elhosseni                                | اسم استاذ المقرر  |

| Student Information معلومات الطالب |  |                |
|------------------------------------|--|----------------|
| Student's Name                     |  | اسم الطالب     |
| ID number                          |  | الرقم الجامعي  |
| Section No.                        |  | رقم الشعبة     |
| Serial Number                      |  | الرقم التسلسلي |

**General Instructions:**

- Your Exam consists of **7 PAGES**
  - You are **NOT** allowed to use any form of notes, textbooks, or electronic devices (such as mobile phones or tablets) during the exam
  - **Normal Calculators are allowed.**
  - Please read each question carefully and answer all parts as instructed.
- عدد صفحات الامتحان **7 صفحات**  
يمنع استخدام أي نوع من الملاحظات أو الكتب أو الأجهزة الإلكترونية (مثل الهواتف المحمولة أو الأجهزة اللوحية و الساعات الذكية) خلال الامتحان  
يسمح باستخدام الآلة الحاسبة العادية فقط  
يرجى قراءة كل سؤال بعناية والإجابة على جميع الأجزاء كما هو موضح

مع تمنياتنا لكم بالتوفيق

Good Luck

هذا الجزء خاص باستاذ المادة

*This section is ONLY for instructor*

| #     | Course Learning Outcomes (CLOs)                                                                                     | Related Question (s) | Points | Final Score |
|-------|---------------------------------------------------------------------------------------------------------------------|----------------------|--------|-------------|
| CLO01 | Present data in charts and tables                                                                                   | Part1                | 2      |             |
| CLO02 | Compute measures of location and measures of variability                                                            | Part2                | 5      |             |
| CLO03 | Acquire basic concepts of probability theory                                                                        | Part3                | 6      |             |
| CLO04 | Distinguish between discrete, continuous random variables and probability distributions                             | Part4                | 7      |             |
| CLO05 | Describe basic concepts of inferential statistics such as sampling distribution of means, the Central Limit Theorem | Part 5               | 4      |             |
| CLO06 | Analyze measures of location and measures of variability                                                            | Part 6               | 2      |             |
| CLO07 | Find the properties of probability distributions.                                                                   | Part 7               | 5      |             |
| CLO08 | Compute probability models for a given problem.                                                                     | Part 8               | 4      |             |
| CLO09 | Implement probability methods to solve real-world problems.                                                         | Part 9               | 2      |             |
| CLO10 | Apply basic concepts of inferential statistics.                                                                     | Part 10              | 3      |             |
| Total |                                                                                                                     |                      | 40     |             |

**Part 1: CLO1**

...../2 MARKS

Q1: A survey was conducted to record the number of cups of coffee consumed daily by 25 employees in an office. The data are as follows: 2, 3, 1, 4, 2, 3, 2, 1, 3, 2, 4, 3, 2, 1, 3, 2, 2, 3, 4, 2, 1, 2, 3, 2, 1

(1) Construct a frequency table for the data?

(2 Mark)

| # Cups | Freq |
|--------|------|
| 1      | 5    |
| 2      | 10   |
| 3      | 7    |
| 4      | 3    |

**Part 2: CLO 2**

...../5 MARKS

Q1: Find the sample standard deviation of the following data set, which represents the scores of a sample of students in a Statistics (STAT) class: 62 61 59 58 60 (Show Details)

A. 1.5      B. 2      C. 2.5      D. 1.58

Q2: What are the quartiles Q2 and Q3 of the following data set of scores?: 13, 26, 15, 5, 23, 7, 14, 36, 53, 30, 42

A. 13, 23      B. 23, 36      C. 7, 23      D. 13, 36

Q3: The sales figures for the first six months of the year at a retail store are as follows (in dollars): 2200, 2500, 2300, 2400, 2200, and 2700. What is the range of the sales figures?

A. 200      B. 400      C. 500      D. 600

**Essay Questions**

(2 marks):

Reem received the following marks on her first 4 math quizzes: 65, 78, 82, and 90. What score must Reem earn on her fifth quiz to achieve an average of 80 across all five quizzes?

$$\text{Total score} = 80 \times 5 = 400$$

$$\begin{aligned} \text{Sum of the first 4 Quiz scores} \\ = 65 + 78 + 82 + 90 = 315 \end{aligned}$$

$$\text{Fifth score} = 400 - 315 = 85$$

**Part 3: CLO 3**

...../6 MARKS

|                                                                                                            |                                     |                                                   |                                |
|------------------------------------------------------------------------------------------------------------|-------------------------------------|---------------------------------------------------|--------------------------------|
| Q4: If events A and B are independent, then:                                                               |                                     |                                                   |                                |
| A. $P(A \cup B) = P(A) + P(B)$                                                                             | B. $P(A \cup B) = P(A) \times P(B)$ | C. $P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$ | D. $P(A \cup B) = P(A) - P(B)$ |
| Q5: If two events are mutually exclusive, what is the probability of both events occurring simultaneously? |                                     |                                                   |                                |
| A. 0                                                                                                       | B. 1                                | C. 0.5                                            | D. it depends on events        |
| Q6: The probability of an event must always be:                                                            |                                     |                                                   |                                |
| A. Greater than 1                                                                                          | B. Between 0 and 1 (inclusive)      | C. Negative if the event is rare                  | D. Exactly equal to 0.5        |
| Q7: If $P(A)=0.4$ and $P(B)=0.3$ and A and B are independent, then $P(A \cap B) =$                         |                                     |                                                   |                                |
| A. 0.12                                                                                                    | B. 0.43                             | C. 0.4                                            | D. 0.7                         |

**Essay Questions**

2 marks):

A university is awarding **three** scholarships: "Academic Excellence", "Leadership", and "Community Service". These scholarships will be given to a group of 8 students, with each student receiving at most one scholarship. How many different ways can the scholarships be assigned to the students?

choose a student for Academic Excellence : 8

" " Leadership : 7

" " Community Service : 6

$$= 8 \times 7 \times 6 = 336$$

Another solution  $\Rightarrow P(8,3) = \frac{8!}{8-3!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$

**Part 4: CLO4**

...../7 MARKS

|                                                                                                                  |                                                               |
|------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------|
| Q8: ..... representing the number of successes occurring in a given time interval or a specified region of space |                                                               |
| A. Normal Distribution                                                                                           | C. Continuou Uniform Distribution                             |
| B. Binomial Distribution                                                                                         | D. Poisson distribution                                       |
| Q9: $Z \sim N(0,1)$ is called                                                                                    |                                                               |
| A. Standard normal distribution                                                                                  | C. Continuou Uniform Distribution                             |
| B. Normal distribution                                                                                           | D. Area under the curve.                                      |
| Q10: Which of the following is an example of a discrete random variable?                                         |                                                               |
| A. The number of phone calls in an hour                                                                          | C. Height of student in the class                             |
| B. The weight of a newborn baby (in kg.)                                                                         | D. The salary of employees                                    |
| Q11 : A probability distribution must satisfy which of the following?                                            |                                                               |
| A. All probabilities are greater than 1                                                                          | C. The sum of probabilities is less than 1.                   |
| B. Probabilities can be negative.                                                                                | D. Each probability is between 0 and 1, and the sum equals 1. |
| Q12: In Binomial Distribution the Variance value is equal to                                                     |                                                               |
| A. $\mu$                                                                                                         | C. $np(1-p)$                                                  |
| B. $np$                                                                                                          | D. None of the above.                                         |

Essay Questions

(2 marks):

A random variable  $X$  follows a binomial distribution with parameters  $n = 10$  and  $p = 0.5$ . What is the probability of getting exactly 3 successes? "Explain the result".

$$n = 10 \quad p = 0.5$$

$$P(X=3) = \binom{10}{3} (0.5)^3 (0.5)^7$$

$$= 0.117$$

← About 12 times out of 100  
You would expect to see exactly 3 successes

Part 5: CLO5

...../4 MARKS

(2 Marks)

Q13: In the context of inferential statistics, what does the term "standard error" refer to?

- |                                              |                                                                           |
|----------------------------------------------|---------------------------------------------------------------------------|
| A. The standard deviation of a sample        | C. The difference between the sample mean and population mean             |
| B. The error margin in a confidence interval | D. The standard deviation of the sampling distribution of the sample mean |

Q14: If a sample of size 64 is drawn from a population with a mean of 32 and a standard deviation of 8, what is the standard error of the sample mean?

- |      |        |
|------|--------|
| A. 8 | C. 0.4 |
| B. 4 | D. 1   |

Q2: A medical researcher is evaluating the effectiveness of a new rehabilitation program designed to shorten patient recovery times after knee surgery. She takes a random sample of  $n = 64$  patients who completed the program and finds that the sample mean recovery time is  $\bar{X} = 15$  days. Assume the population standard deviation ( $\sigma$ ) is known to be 4 days. Using a 95% confidence level (with  $Z_{\alpha/2} = 1.96$ ), what is the interval best represents the 95% confidence interval for the population mean recovery time? (2 Marks)

$$n = 64 \quad \bar{X} = 15 \quad \sigma = 4 \quad \text{Confidence level: } 95\%$$

$$\text{Critical value } Z_{\alpha/2} = 1.96$$

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{64}} = 0.5$$

$$\text{Margin of error} = Z_{\alpha/2} \times SE = 1.96 \times 0.5 = 0.98$$

$$\text{Confidence Interval } \bar{X} \pm \text{Margin} = (14.02, 15.98)$$

Part 6: CLO6

...../2 MARKS

Q1: Consider the following dataset representing the ages (in years) of 6 people:

25, 30, 27, 32, 28, 35.

- Task: Calculate the range, the median, variance, and standard deviation of this dataset

Arrange in order  $\rightarrow 25, 27, 28, 30, 32, 35$

$$\text{Range} = 10$$

$$\text{Median} = 29$$

$$\bar{X} = 29.5$$

$$S^2 = 13.1$$

$$S = 3.62$$

Part 7: CLO7

...../5 MARKS

Choose the correct answer

(3 marks)

Q15: What is the relationship between the mean ( $\mu$ ) and the variance of a Poisson distribution?

- |                                                  |                                                           |
|--------------------------------------------------|-----------------------------------------------------------|
| A. The variance is always greater than the mean. | C. The variance is always less than the mean.             |
| B. The variance is always equal to the mean.     | D. There is no relationship between the mean and variance |

Q16: Two variables .... and ... control the shape of the normal curve, suppose there are two distributions

- |                      |                                    |
|----------------------|------------------------------------|
| A. mean and range    | C. standard deviation and variance |
| B. mean and variance | D. mean and standard deviation     |

Q17: In a continuous probability distribution, the probability of any single point is:

- |      |                               |
|------|-------------------------------|
| A. 0 | C. 0.5                        |
| B. 1 | D. It depends on the question |

Q18: The following formula  $E(x) = \sum x f(x) = \sum x \frac{1}{B-A}$  is used to compute?

- |                                  |                                 |
|----------------------------------|---------------------------------|
| A. Mean of Binomial Distribution | C. Mean of Poisson Distribution |
| B. Mean of Uniform Distribution  | D. Mean of Normal Distribution  |

Q19: What is the expected (mean) number of defective parts in a batch of 10 with  $p = 0.05$ ?

- |         |         |
|---------|---------|
| A. 0.01 | C. 0.5  |
| B. 0.1  | D. 0.05 |

Q20: A bag contains 3 red balls and 5 blue balls. What is the probability of randomly picking a red ball?

- |        |        |
|--------|--------|
| A. 1/5 | C. 3/5 |
| B. 5/8 | D. 3/8 |

Essay Questions

(2 marks):

An industrial quality control engineer is inspecting a batch of 10 identical machine parts produced in a manufacturing line. The probability that any single part is defective is  $p = 0.05$ . Assuming each part's condition is independent of the others. What is the probability that no parts are defective (i.e.,  $X = 0$ )?

$$n = 10 \quad p = 0.05$$

$$P(X=0) = \binom{10}{0} (0.05)^0 (0.95)^{10}$$

$$\approx 0.5987$$

# Part 8: CLO 8

...../4 MARKS

Q1: Suppose that a chips box has 25% of the bags are winner bags, the child's mom lets him select three bags of the box. The child want to know the probability of ( $x$  follow Binomial distribution): (2 marks)

- Getting no winner bags
- Probability of getting at least two winner bags.

(show all steps)

$$p = 0.25 \quad n = 3$$

$$P(x=0) = \binom{3}{0} (0.25)^0 (0.75)^3 = 0.4219$$

$$P(x \geq 2) = P(x=2) + P(x=3)$$

$$P(x \geq 2) = 0.1563$$

Q2: Using the standard normal table above, find the area under the curve that lies:

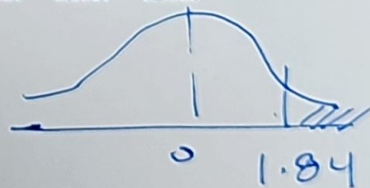
(2 marks)

- To the right of  $Z=1.84$
- $P(-1.2 \leq Z \leq 2.1)$

Table A.3 (continued) Areas under the Normal Curve

| z   | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5676 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

[1]  $P(Z \leq 1.84) = 0.9671$   
 $P(Z > 1.84) = 0.0329$



[2]  $P(-1.2 \leq Z \leq 2.1) = P(Z \leq 2.1) -$   
 $(1 - P(Z \leq 1.2))$   
 $= 0.8670$

Part 9: CLO9

...../2 MARKS

The piece is manufactured in a factory and directed to test its validity by three workers. If the probability of dealing by the first worker is 0.3, second worker is 0.4 and third worker is 0.3.

- If it valid and chosen by first worker is 0.84.
  - If it valid and by second worker is 0.88.
  - If it valid and chosen by first worker is 0.85.
- 1) What is the probability of that a selected random product is valid?
  - 2) Probability the product was inspected by the first worker, given it is valid? (You should show details answer)

$$\begin{aligned} \boxed{1} \quad p(v) &= p(w_1)p(v|w_1) + p(w_2)p(v|w_2) + p(w_3)p(v|w_3) \\ &= (0.3)(0.84) + (0.4)(0.88) + (0.3)(0.85) \\ &= 0.859 \end{aligned}$$

$$\begin{aligned} \boxed{2} \quad p(w_1|v) &= \frac{p(w_1)p(v|w_1)}{p(v)} \\ &= \frac{0.3 \times 0.84}{0.859} = 0.293 \end{aligned}$$

PART 10: CLO10:

...../3 MARKS

A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution. (2 marks)

- (a) What is the probability density function of X?
- (b) What is the probability that the individual waits more than 6 minutes (at least 6 minutes)?

(show all steps)

$$\begin{aligned} \textcircled{a} \quad X &\sim \text{uniform}(0, 10) \\ \text{PDF} \Rightarrow f(x) &= \begin{cases} \frac{1}{10} & 0 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\textcircled{b} \quad p(x \geq 6) = \frac{10-6}{10-0} = \frac{4}{10} = 0.4$$

the margin of error

Q1: A Maximum error estimation is ..... and calculated by the

Equation  $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

(1 marks)