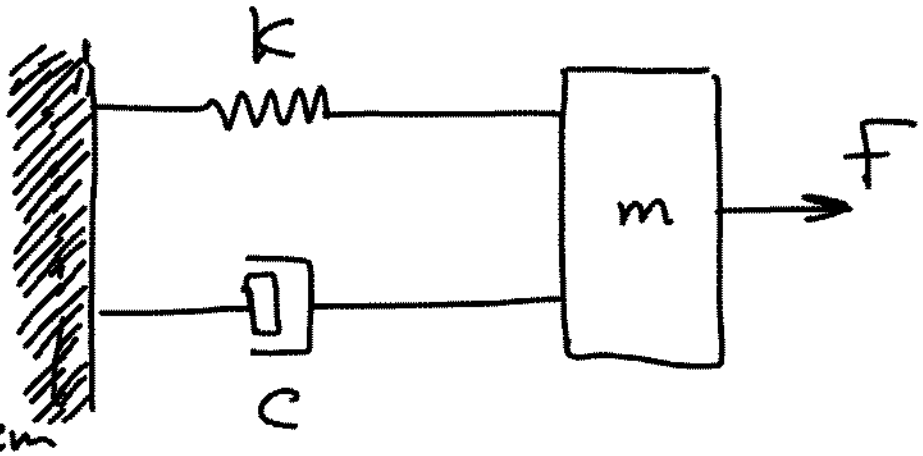
Mechanical System

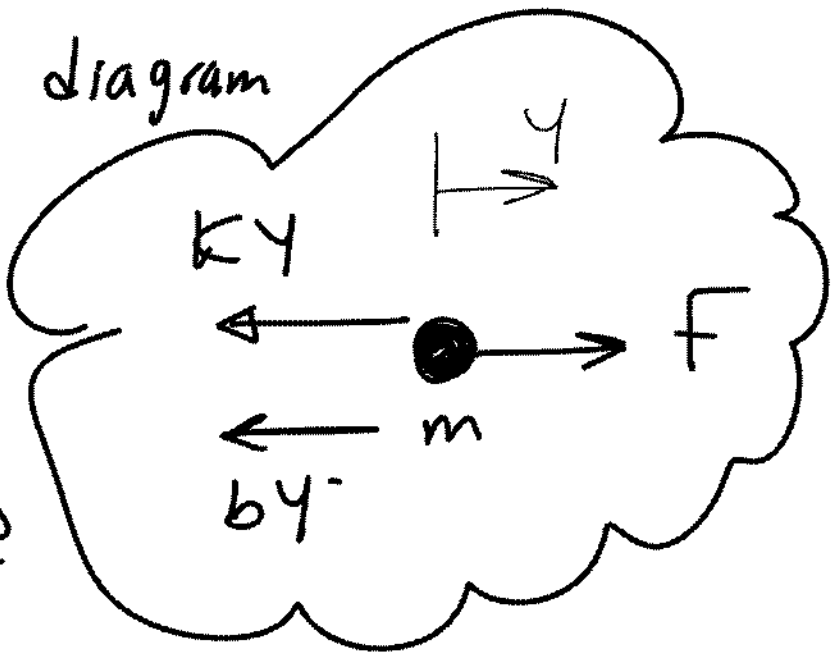
Ex

Fig. next shows a simple mechanical translational system



with a mass, a spring, and a dashpot. A force F is applied to the system. Derive a mathematical model for the system

Sol $\Rightarrow$ Free-body diagram



$$f - b\dot{y} - ky = m\ddot{y}$$

$$m\ddot{y} + b\dot{y} + ky = f$$

$$(ms^2 + bs + k)Y = F$$

$$\frac{Y}{F} = \frac{1}{ms^2 + bs + k}$$

Assume $f(t) = u(t) = 1 \ N$

$$f(t) = u(t) \Rightarrow F(s) = 1/s$$

$$m = 1 \ , \quad b = 0.1 \ , \quad k = 0.1$$

$$\frac{Y}{F} = \frac{1}{s^2 + 0.1s + 0.1}$$

(1) analytically

$$Y = \frac{1}{s(s^2 + 0.1s + 0.1)}$$

$$Y = \frac{10}{s} + \frac{As + B}{s^2 + 0.1s + 0.1}$$

$$\frac{1}{s(s^2+0.1s+0.1)} = \frac{10}{s} + \frac{As+B}{s^2+0.1s+0.1}$$

Coefficient of $s^2 = 0 = 10 + A \Rightarrow A = -10$

Coefficient of $s = 0 = 1 + B \Rightarrow B = -1$

$$\therefore Y = \frac{10}{s} - \frac{10s+1}{s^2+0.1s+0.1}$$

$$Y = \frac{10}{s} - \frac{10(s+0.05)+0.5}{(s+0.05)^2+0.0975}$$

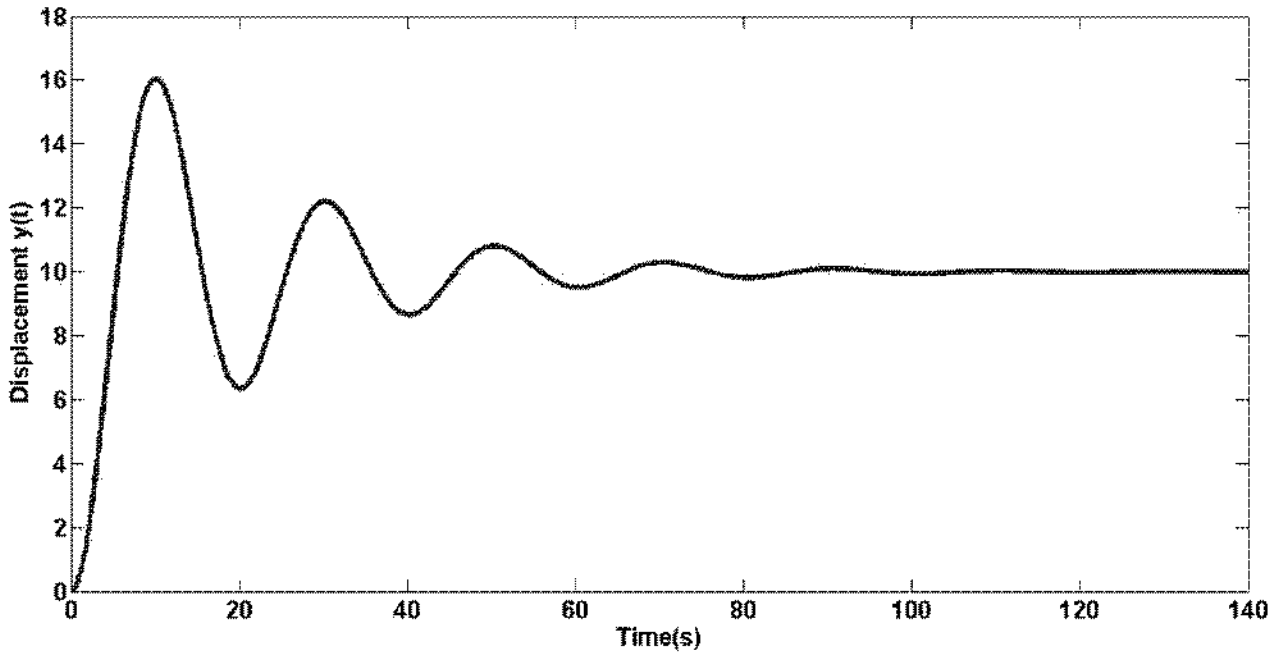
$$\Downarrow$$

$$y(t) = 10 - 10\, e^{-0.05t} \cos\sqrt{0.0975}\, t - \frac{0.5}{\sqrt{0.0975}} e^{-0.05t} \sin\sqrt{0.0975}\, t$$

$$\Downarrow$$ (2) by MATLAB .m code

$$\frac{Y}{F} = \frac{1}{s^2 + 0.1s + 0.1}$$

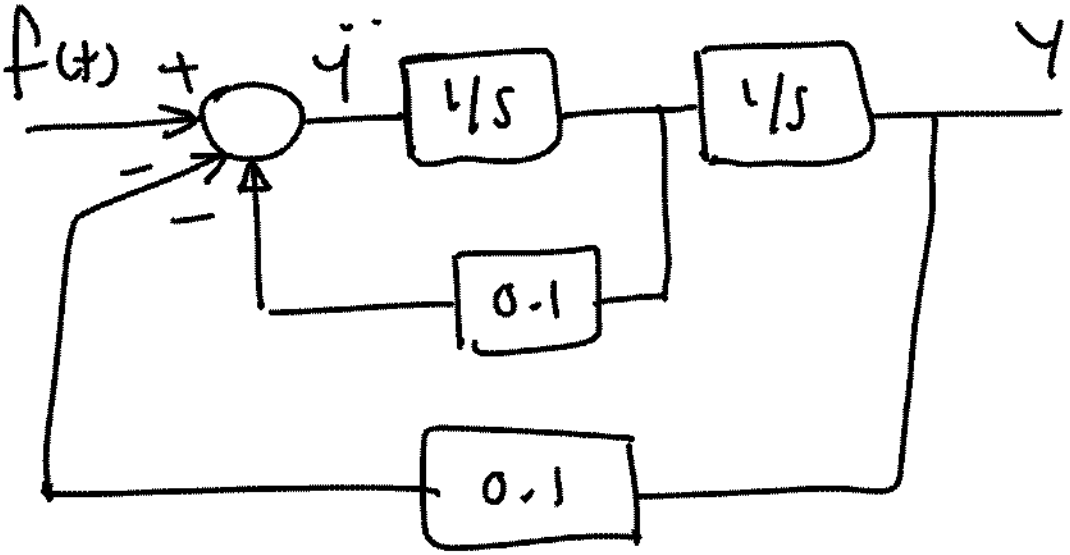
```
m = 1 ;
b = 0.1 ;
K = 0.1 ;
num = [1];
den = [m ▽ b ▽ K]
sys = tf (num, den);
step (sys) ;
```

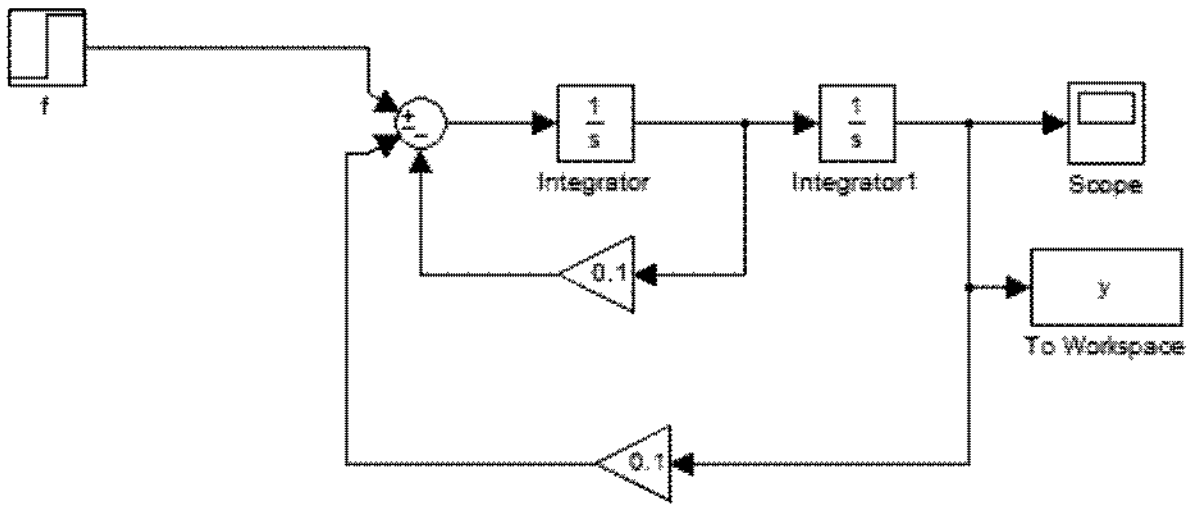


③ MATLAB // SIMULINK

$$\frac{Y}{F} = \frac{1}{s^2 + 0.1s + 0.1}$$

$$\ddot{y} + 0.1\dot{y} + 0.1y = f(t)$$





Ex ⇒

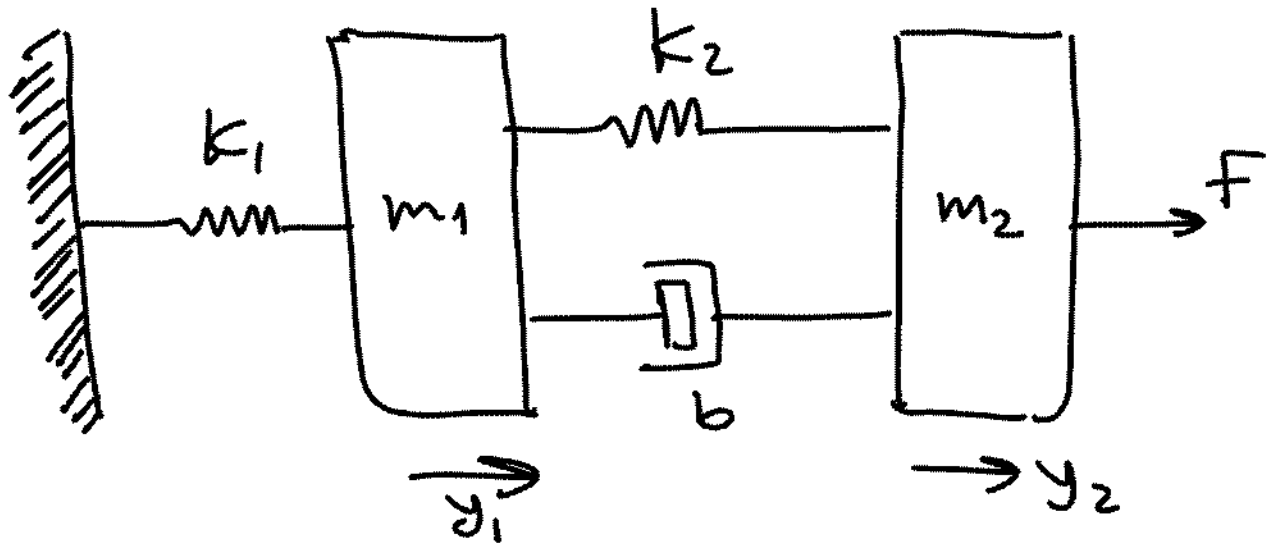
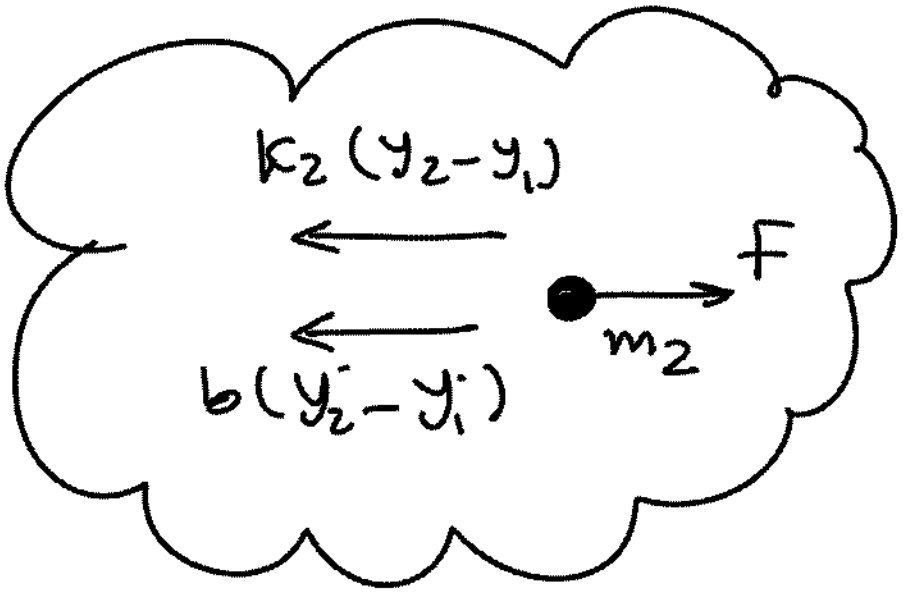


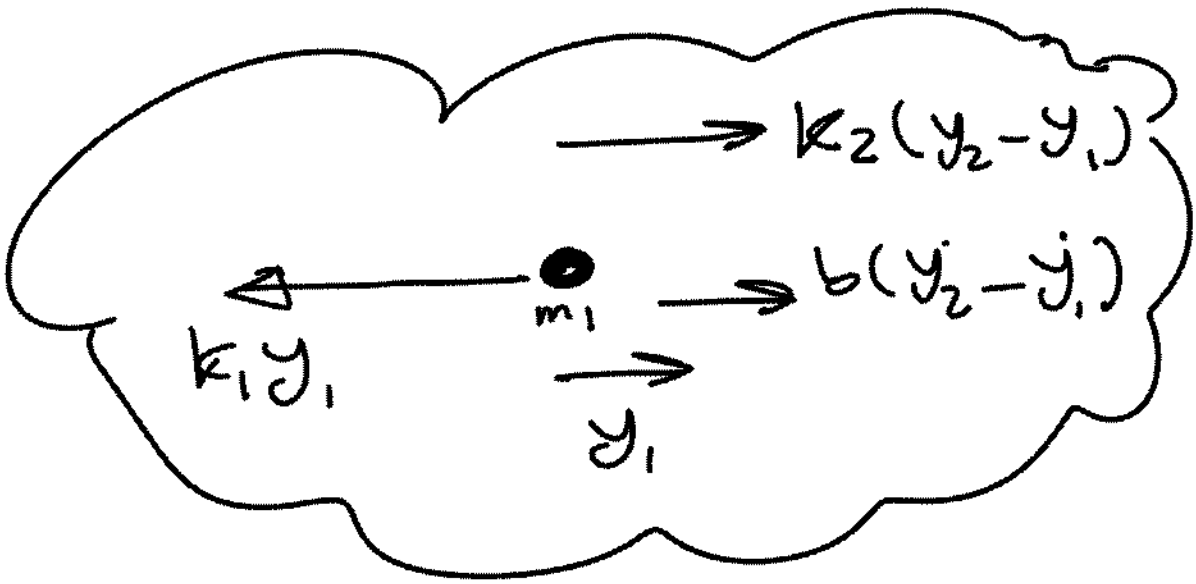
Fig. above shows a mechanical system with two masses and two springs. Obtain the mathematical model of the system.

Sol.



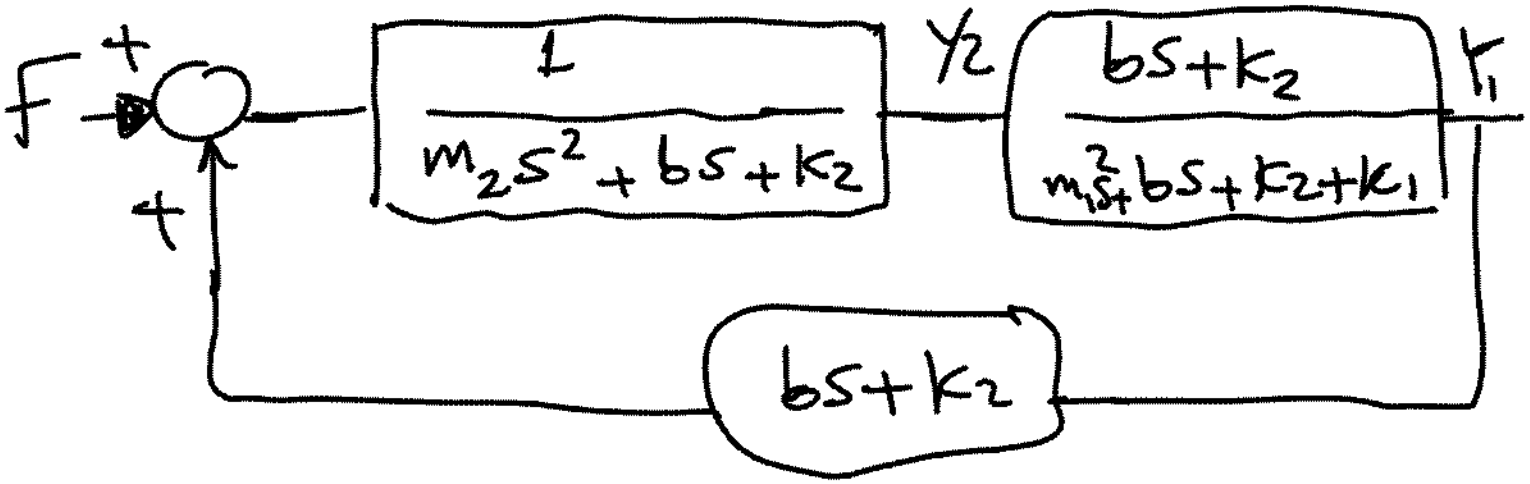
$$f - b(\dot{y}_2-\dot{y}_1) - k_2(y_2-y_1) = m_2\ddot{y}_2$$

$$F = (m_2s^2 + bs + k_2)Y_2 - (bs+k_2)Y_1$$



$$b(\dot{y}_2-\dot{y}_1)+k_2(y_2-y_1)-k_1y_1=m_1\ddot{y}_1$$

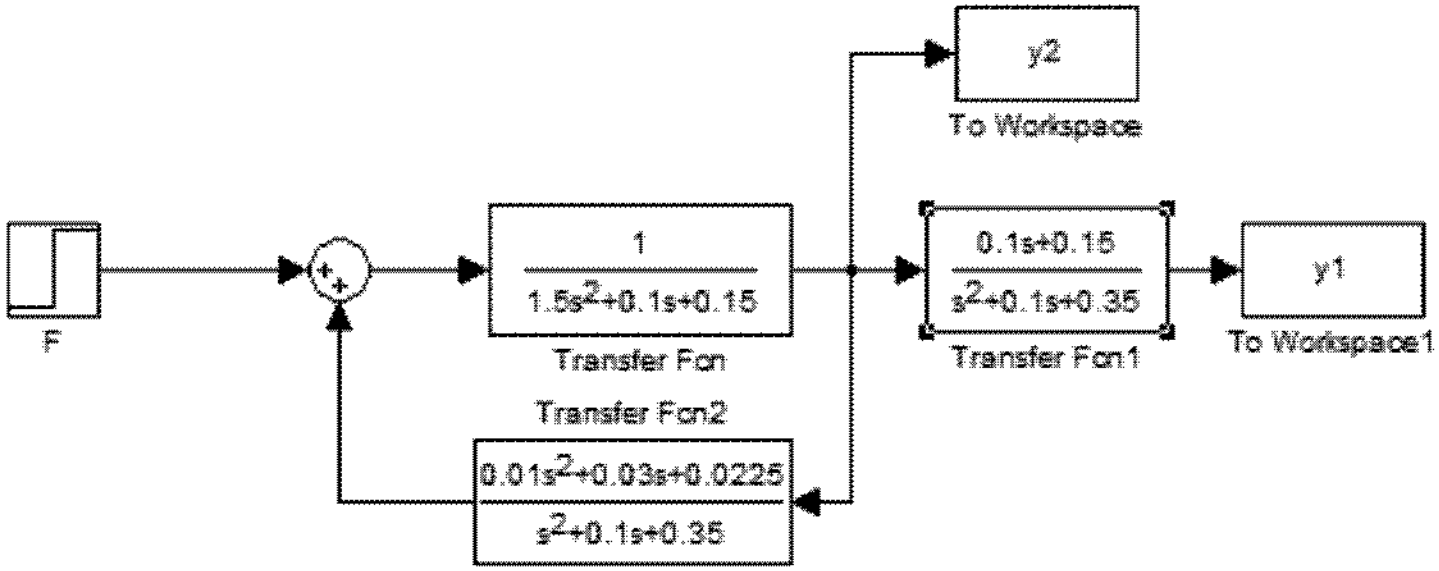
$$(bs+k_2)Y_2=(m_1s^2+bs+k_2+k_1)Y_1$$

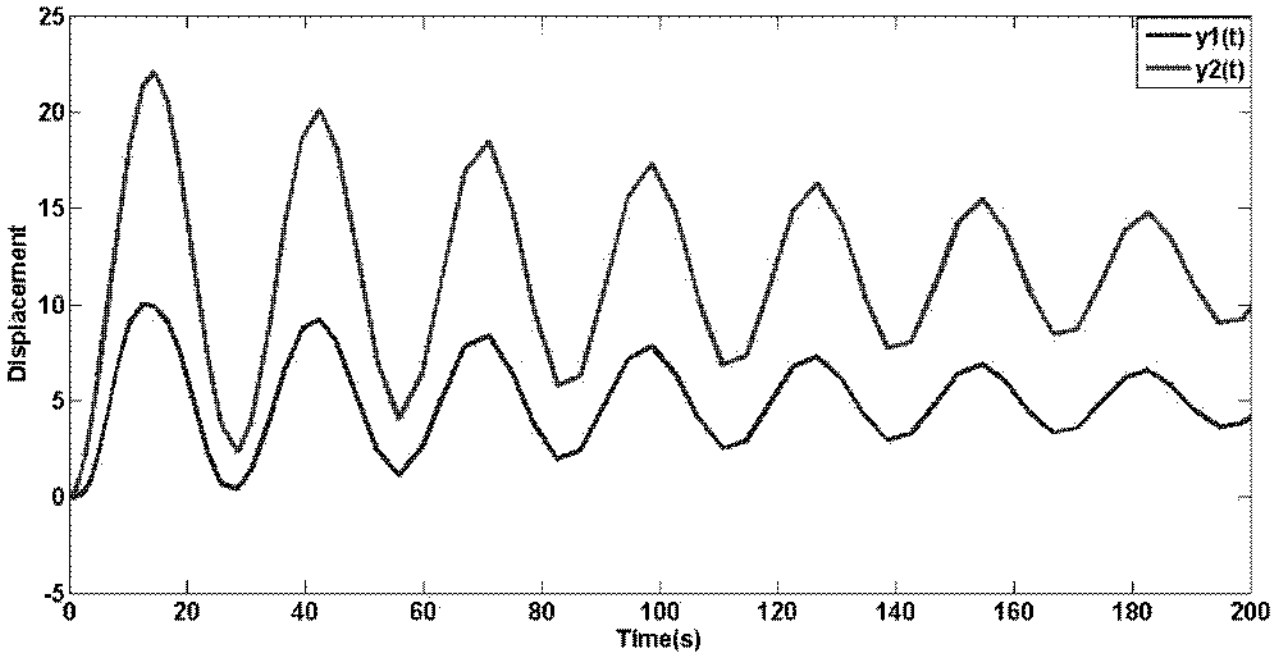


Assume

$F=1$, $m_1=1$, $m_2=1.5$, $b=0.1$

$k_1=0.2$, $k_2=0.15$


y2
To Workspace
F
1
1.5s2+0.1s+0.15
Transfer Fcn
0.1s+0.15
s2+0.1s+0.35
Transfer Fcn1
y1
To Workspace1
Transfer Fcn2
0.01s2+0.03s+0.0225
s2+0.1s+0.35


y1(t)
y2(t)
Displacement
Time(s)
25
20
15
10
5
0
-5
0
20
40
60
80
100
120
140
160
180
200